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Naming and Epistemic Necessity

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Abstract: Kripke (1980) hypothesized a link between rigidity and scope: a singular term is rigid over a space S of possibilities just in case it is scopeless with respect to modals that quantify over S. Kripke's hypothesis works well when we consider the interaction of singular terms with metaphysical modals, but runs into trouble when we consider the interaction of singular terms with epistemic modals. After describing the trouble in detail, and considering one non-solution to it, we develop a novel version of dynamic semantics that resolves the problem.

1 Introduction

One of the central theses of *Naming and Necessity* is that proper names are rigid designators, while (most ordinary) definite descriptions are not. Let us remind ourselves of one of Kripke's arguments for this two-fold claim (Kripke 1980, 48–49).

Consider the contrast between the following pair of sentences:

(1) Trump might not have been Trump.

(2) The President of the U.S. in 2018 might not have been the President of the U.S. in 2018.

The modal here is to be understood as expressing metaphysical possibility. The first of these appears to be unambiguously false, for it seems that no one other than Trump could have been Trump. But things are different with the second sentence. For while it does have a false reading, it also appears to have a true reading. The latter can be brought out by saying,

> 'The person who is in fact the President in 2018 – that man – *he* might not have been the President in 2018.'

The parallel speech with 'Trump' replacing 'President in 2018' is harder to make sense of.

The above contrast between (1) and (2) is elegantly explained by Kripke's hypothesis that proper names are rigid designators while (most ordinary) definite descriptions are not. Let us agree to call both proper name and definite description 'singular terms.' If we ignore the possibility that a singular term might fail to refer at some possible worlds, then we can say:

Definition 1. A singular term a is *rigid* just in case: for any pair of possible worlds v, v', the referent of a at v is identical to the referent of a at v'.

The explanation of the contrast between (1) and (2) then proceeds as follows. Let us suppose that each of the above sentences has two different "logical forms," a *de dicto* form and a *de re* form. The *de dicto* forms of (1) and (2) correspond (respectively) to the following English sentences:

> It might have been that [Trump is not Trump].
>
> It might have been that [the President of the U.S. in 2018 is not the President of the U.S. in 2018].

These *de dicto* forms may (respectively) be formalized as follows (using t to translate 'Trump' and p to translate 'the President of the U.S. in 2018'):

$$\Diamond_m t \neq t$$

$$\Diamond_m p \neq p$$

(The subscript 'm' here indicates that the modality in question is metaphysical.) These *de dicto* logical forms are both false. For whether or not a is a rigid designator, there is no possible world at which 'a is not a' is true.

So the contrast between (1) and (2) must emerge when we consider their *de re* forms. The *de re* form of (1) corresponds to the following English sentence:

> Trump is such that it might have been that [he was not Trump]

and may be formalized with the help of an "abstraction operator" as follows:

$$(\lambda x. \Diamond_m x \neq t)(t).$$

This logical form is evaluated for truth by first finding the referent r of 'Trump' at the actual world v and then asking whether there is a possible world v' at which r is distinct from the referent r' of 'Trump' at v'. On (and only on) the assumption that proper names like 'Trump' are rigid, there is no such world. For if 'Trump' is rigid, then the referent of 'Trump' at any other world is that very same man r. So there will be no possible world v' at which the referent of 'Trump' at v' is distinct from the actual referent of 'Trump' – for there is no possible world at which r is distinct from r.

The *de re* form of (2), on the other hand, is predicted to be *true* given the assumption that the description 'the President of the U.S. in 2018' is non-rigid. The *de re* form of this sentence corresponds to the following English sentence:

> The President of the U.S. in 2018 is such that it might have been that [he was not the President of the U.S. in 2018]

and may be formalized as follows:

$$(\lambda x.\Diamond_m x \neq p)(p).$$

Again, we evaluate this form by first finding the referent r of 'the President of the U.S. in 2018' in the actual world v and then asking whether there is a possible world v' in which r is distinct from what 'the President of the U.S. in 2018' refers to in v'. If that description is non-rigid, then there *must* be such a world v'. For if 'the President of the US in 2018' is non-rigid, then there must be a world v' at which it refers not to its actual referent (Trump) but to some other individual (Hillary Clinton, say). For if there were no such world, then 'the President of the U.S. in 2018' would refer to its actual referent in every world, and so would be rigid.

Thus, if the proper name 'Trump' is rigid, both the *de dicto* and *de re* forms of (1) are false. And if the description 'the President of the U.S. in 2018' is non-rigid, then the *de re* form of (2) is true. So this account predicts the apparent asymmetry between (1) and (2).

Some may wish to take issue with some aspect of the foregoing argument; I do not. Instead, what I want to point out is that this sort of reasoning leads us into a puzzle when we move from considering metaphysical modals (Kripke's concern) to *epistemic modals*, the topic of much recent discussion in the philosophy of language.[1]

Let me begin by stating a generalization suggested by the foregoing considerations. Let's say that a singular term is *scopeless* with respect to a modal operator M just in case it doesn't matter whether you interpret it inside or outside the scope of M. More precisely, we can say that:

> A singular term a is *scopeless with respect to modal* M iff for all formulas ϕ, $(\lambda x.M\phi)(a)$ is equivalent to $M\phi(a/x)$, where $\phi(a/x)$ is the result of replacing all free occurrences of x in ϕ with a.

So to say that a term a is scopeless with respect to M is to say that a *de re* modal predication featuring a and M is equivalent to its *de dicto* counterpart: the *de re*-*de dicto* distinction, as it pertains to a and M, collapses.

Kripke's argument seems to suggest the following:

> SCOPELESSNESS ⇔ RIGIDITY
>
> A singular term a is scopeless with respect to a modal operator M just in case a is rigid over the domain over which M quantifies.

[1] Kripke is explicit in various plans that epistemic modals of the sort we're discussing fall outside the scope of his inquiry. See, for example, Kripke (1980, 103, 141, 143).

Proper names are scopeless with respect to metaphysical modals, a fact seemingly explained by their rigidity over the space of metaphysically possible worlds. Ordinary definite descriptions are *not* scopeless with respect to such modals, a fact seemingly explained by their metaphysical *non*-rigidity.[2]

Note that I have stated the above generalization not using the notion of "rigidity full-stop," as it were, but with a more generalized notion of "rigid over" a set of worlds. If we continue to ignore the possibility that a singular term might fail to refer at some worlds, this notion can be defined as follows:

Definition 2. A singular term a is *rigid over a set S of worlds* just in case, for any worlds v, v' in S, the referent of a in v is identical to the referent of a in v'.

The notion of rigidity Kripke had in mind can then be recovered as follows: a singular term is rigid in Kripke's sense just in case it is rigid over the set of metaphysically possible worlds.

To see what issue epistemic modals raise in this context, consider the epistemic counterpart of sentence (2):

(3) The inventor of bitcoin might not be the inventor of bitcoin.

Unlike sentence (2), it is difficult to hear a true reading of sentence (3), as Aloni (2001, Ch. 3) and Yalcin (2015) observe. Obviously, the *de dicto* form of this sentence will be false. But even the *de re* form seems to be false. To see this, consider the inventor of bitcoin, whoever that may be. Is it true that he or she might not be the inventor of bitcoin? How could that be? That person is, after all, given to us *as* the inventor of bitcoin.

I'm inclined to agree with Aloni and Yalcin that sentences like (3) are unambiguously unacceptable. But if you feel uncertain about this, it might help to consider one of the following instead:

(4) The inventor of bitcoin might turn out not to be the inventor of bitcoin.

(5) We might discover that the inventor of bitcoin is not the inventor of bitcoin.

It's hard for me to hear a true reading of either of these. Again, any reasonable theory will predict that the *de dicto* reading of (5), for example, is false. So focus on the *de re* reading. And consider the inventor of bitcoin, whoever that may be. Might we discover that that person is not, in fact, the inventor of bitcoin? How could we? Whatever course our inquiry takes, we can be sure that it will not lead us to the discovery that that person is not the inventor of bitcoin.

Now, when we move from metaphysical to epistemic modals, we have to deal with the fact that the truth or acceptability of a sentence containing an epistemic

[2]Kripke makes the connection between scopelessness and rigidity in various places; see, for example, (Kripke 1980, 12). For a treatment of rigidity and scope in modal logic, see Fitting and Mendelsohn (1998, §10.2).

modal is not absolute. Precisely how to understand this fact has been the subject of much recent dispute between *contextualists* and *relativists*, but I don't think those controversies have much bearing on the present issue, for the following reason.[3] It is reasonable for us to proceed by framing the issue in terms of whether or not our target sentences are *assertable* in our present context. In our context c, (6) is assertable, but neither (3) nor (7) is assertable in c. But the disputes in question tend not to concern the assertability of sentences containing epistemic modals, but the conditions under which they may be appropriately disputed or retracted. And as MacFarlane (2014, Ch. 10) argues, contextualists and relativists can agree that assertability is to be evaluated in terms of the notion of "truth at a context" So we should be safe in appealing to that notion, at least in our initial formulation of these issues.

Now we are assuming that (3) has no true reading in our context c. Any reasonable theory will predict that the *de dicto* form of (3) has no true reading in our context. But consider the *de re* form of (3):

$$(\lambda x.\Diamond_e x \neq b)(b)$$

(Here, b translates 'the inventor of bitcoin,' and the subcript 'e' indicates that the modal is epistemic.) The falsity of this form in our context suggests that the definite description 'the inventor of bitcoin' is scopeless with respect to the the epistemic modal 'might.' Thus, if we extend Kripke's line of reasoning – embodied in the principle SCOPELESSNESS $\Leftrightarrow$ RIGIDITY – we are led to the conclusion that that description is rigid over $\mathcal{B}_c$, the domain over which epistemic 'might' quantifies in c. For suppose it were not rigid over $\mathcal{B}_c$. Then if r is the referent of 'the inventor of bitcoin' in the actual world v, there would have to a world v' in $\mathcal{B}_c$ at which that description referred to someone other than r. But such a world would then underwrite the truth of (3) at our context.

So it would seem that, while being *metaphysically non-rigid*, ordinary definite descriptions are nevertheless *epistemically rigid in c* – rigid over the set of worlds over which epistemic 'might' quantifies in c. This is just Kripke's argument from scope-sensitivity to non-rigidity, adapted to the epistemic case.

But this leads to a problem. For isn't it obvious that the definite description 'the inventor of bitcoin' is epistemically non-rigid in our present context? Here is a fact about bitcoin: no one knows who invented it.[4] But technology journalists and other interested parties have a few names on their list of suspects. One such suspect is Elon Musk, CEO of Tesla; another is an Australian computer scientist named 'Craig Steven Wright.' So it would seem that, in some worlds compatible with what we know, 'the inventor of bitcoin' refers to Elon Musk; in others, to Craig Steven Wright. Thus, it

[3]For discussion of this debate, see Egan *et al.* 2005, Egan 2007, Stephenson 2007, Yalcin 2007, 2011, von Fintel and Gillies 2008, 2011, Dowell 2011, MacFarlane 2011, 2014, and Schaffer 2011, among others. Related issues are discussed in Hacking 1967 and DeRose 1991.

[4]Well, *someone* (e.g. the inventor) presumably knows, but his or her identity is not (as of mid-2018) a matter of public knowledge.

seems that the definite description 'the inventor of bitcoin' is *non-rigid* over the set of worlds $\mathcal{B}_c$ over which 'might' quantifies in our present context.

The claim that 'the inventor of bitcoin' is non-rigid over $\mathcal{B}_c$ is also supported by the fact that the following sentence appears to be true in our context:

(6) The CEO of Tesla might be the inventor of bitcoin, but (then again) the CEO of Tesla might not be the inventor of bitcoin.

$\Diamond_e o = b \wedge \Diamond_e o \neq b$

(Here o translates 'the CEO of Tesla.') For if this sentence is true in our context c, then at least one of the two descriptions that figure in that sentence must be non-rigid over the class of worlds over which 'might' quantifies in c. To see this, suppose, for *reductio*, that (6) is true relative to c and that both o and b are epistemically rigid over $\mathcal{B}_c$. Then since $\Diamond_e o = b$ is true at c, there is a world $w \in \mathcal{B}_c$ at which o and b pick out the same object r. But then since o and b are rigid over $\mathcal{B}_c$, it follows that, for any world $w' \in \mathcal{B}_c$, the referent of o at w' will be r and the referent of b at w' will be r. Thus, $o \neq b$ will be false at w'. Since this holds for an arbitrary world in $\mathcal{B}_c$, it holds for them all, which means $\Diamond_e o \neq b$ is false at c. But this contradicts the supposition that (6) is true at c.

Of course, this argument only shows that *one* of the two descriptions in (6) is non-rigid over $\mathcal{B}_c$. So we could in principle square this result with the claim that 'the inventor of bitcoin' is rigid over $\mathcal{B}_c$ by maintaining that 'the CEO of Tesla' is non-rigid over that class of worlds. But this fails to avoid the problem, since it also appears that (7) is unambiguously false in our context:

(7) The CEO of Tesla might not be the CEO of Tesla.

Kripke-style considerations will again lead us from this observation to the conclusion that 'the CEO of Tesla' is rigid over $\mathcal{B}_c$, and so our problem returns.

So the unambiguous falsity of (3) and (7) in our context suggest that the descriptions figuring in those sentences are both epistemically *rigid* in our context. But the apparent truth of (6) in our contexts suggests instead that at least one of those descriptions is epistemically *non-rigid* in our context.

This puzzle concerns definite descriptions, but essentially the same puzzle also arises in connection with proper names. To see this, consider the following sentence:

(8) Elon Musk might be Satoshi Nakamoto, but (then again) Elon Musk might not be Satoshi Nakamoto.

$\Diamond_e m = n \wedge \Diamond_e m \neq n$

(Here m translates 'Elon Musk' and n 'Satoshi Nakamoto.') More news about bitcoin: 'Satoshi Nakamoto' is the name used by the unknown person who created bitcoin. That is, the person who invented bitcoin has published whitepapers and blog posts under that name, but it is not a matter of public knowledge which legal person is picked out by that name. So it seems to me that (8) is true in our present context, since (a) we don't know who Satoshi Nakamoto is, nor is it a matter of public knowledge who he or she is, and (b) Elon Musk is not a wholly implausible candidate for being Satoshi Nakamoto.

For reasons parallel to the ones given above in our discussion of (6), the truth of (8) in our present context c suggests that at least one of the two names that figure in that sentence is non-rigid over $\mathcal{B}_c$. But that result seems to be in tension with the fact that the following sentences appear to be unambiguously false (in our context):

(9) Elon Musk might not be Elon Musk.

(10) Satoshi Nakamoto might not be Satoshi Nakamoto.

For the falsity of the *de re* forms of (9) and (10) in our context suggests that these two names are both rigid over $\mathcal{B}_c$.

Now one might try to get around the proper name version of this puzzle by denying that the correct response to the truth of (8) in c is to conclude that one of the proper names in question is non-rigid over $\mathcal{B}_c$. "Millians" about proper names would presumably maintain that both of those names are rigid over $\mathcal{B}_c$.[5] This would allow them to give a straightforward, Kripkean explanation of the unambiguous falsity of (9) and (10). But such a move would then require giving an alternative explanation of the apparent truth of (8) in the present context. I won't pursue this strategy further here, for even if it were carried off successfully, it would leave untouched our puzzle about definite descriptions, and so the general problem would still be with us.

In what follows, I use the foregoing puzzles to examine theories of modals, singular terms, and variables. I begin in §2 by looking at a theory according to which variables range over *individual concepts* rather than over individuals. Although this theory does resolve the puzzle about definite descriptions, it does so only by upending Kripke's account of sentence (2). That highlights an important constraint on an adequate solution to the puzzle: it should be compatible with Kripke's initial observations about sentences (1) and (2). In §3, I develop a version of *dynamic semantics* that does just this. The version of dynamic semantics developed here builds on earlier work (Groenendijk *et al.* 1996), but also contains a number of innovations. In particular, it shows how to extend the dynamic approach to languages containing an abstraction operator and a metaphysical possibility modal.[6]

[5]For defenses of Millianism, see Salmon (1986) and Soames (2002), among others.

[6]Yalcin (2015) also uses the infelicity of sentences like (3), along with other data, to motivate a version of dynamic semantics similar to the one presented here. But Yalcin's focus is on the interaction between epistemic modals and quantifiers, and he treats definite description along the quantificational model of Russell (1905). Thus, questions about the rigidity and scope of singular terms – questions that constitute the topic of the present essay – do not arise for Yalcin. Also, Yalcin does not discuss metaphysical modals in detail, whereas metaphysical modality has an important role to play in our discussion. Formally, these differences reveal themselves in the fact that, unlike Yalcin's semantics, our semantics is defined for a language that includes individual constants, an

2 Individual concepts

Let's assume that we're not going to abandon the epistemic non-rigidity of descriptions like 'the inventor of bitcoin.' This allows us to maintain that (6) is true in our context. Then the basic trouble is to understand how (3) might be unambiguously false in our context despite the fact that 'the inventor of bitcoin' is non-rigid over $\mathcal{B}_c$. And since the *de dicto* logical form of (3) will be predicted false (in any context) by almost any theory, the trouble really boils down to understanding how the *de re* logical form of (3)

$$(\lambda x.\Diamond_e x \neq b)(b)$$

can be false in our context, given that 'the inventor of bitcoin' is non-rigid over $\mathcal{B}_c$.

Now, the standard way of evaluating this logical form with respect to our context c would be to find the referent r of b at the actual world v, and then check to see whether there is a world $v' \in \mathcal{B}_c$ at which r is distinct from the referent r' of b at v. Note that according to this procedure for evaluating $(\lambda x.\Diamond_e x \neq b)(b)$, the λ-abstract $(\lambda x.\Diamond_e x \neq b)$ is, so to speak, 'looking for' an individual. The individual we hand it in the course of our evaluation is the individual to whom b refers in the actual world. If this is Elon Musk, then we are handing that λ-abstract Elon Musk. The resulting logical form is then predicted to be true in our context since, for all we know, Elon Musk is not the inventor of bitcoin.

Now, a tempting thought is that, rather than handing that λ-abstract the *referent* or *extension* of b at the actual world, we should hand it the *sense* or *intension* of b. Formally, we can think of the intension of b as an *individual concept*, a function from worlds to individuals. Since b translates 'the inventor of bitcoin,' the intension of b is a function i_b that maps each world w to the inventor of bitcoin at w.[7] We could then assess the logical form by finding, not the extension r of b at the actual world, but the *intension* i_b of b, and then checking to see whether there is a world $v' \in \mathcal{B}_c$ such that i_b maps v' to an individual that is distinct from the referent of b at v'. And we know that there will be no such world. This is because, for each world v', the intension of b maps v' to the referent r' of b at v' – this is just what it means to be the intension of b. Thus, such a theory will predict that the *de re* logical form of (3) is false at every context; it will thus predict the unambiguous falsity of (3).

Note that this argument makes no assumption as to whether or not b is rigid across $\mathcal{B}_c$. This is because the referent r of b at the actual world never enters into

abstraction operator, and a metaphysical possibility modal.

The discussion of Aloni (2001, Ch. 3) is also relevant, though she uses sentences like (3) not to motivate dynamic semantics *per se*, but to constrain her theory of quantified epistemic modality. And like Yalcin's, Aloni's semantics is not defined for a language that contains an abstraction operator, and a metaphysical possibility modal. I discuss Aloni's approach in more detail in AUTHOR (XX).

[7] We continue to ignore the possibility that some worlds may fail to have a unique inventor of bitcoin.

our evaluation of the formula. Thus, whether there is a world v' in $\mathcal{B}_c$ at which the referent r' of b is distinct from r is simply irrelevant to the truth value of the formula.

One way to work this idea out out in detail is to have variables range over individual concepts rather than over individuals. Such systems are often called 'systems of contingent identity' and trace back to Carnap (1947).[8] On this sort of approach, a variable assignment maps each variable to an individual concept, rather than to an individual. So if g is a variable assignment, x a variable, and v a world, $g(x)$ is an individual concept, and $g(x)(v)$ an individual.

Given a variable assignment g and an individual constant a, let $g[x/i_a]$ be that variable assignment which maps x to i_a (the intension of a) and maps each variable y distinct from x to $g(y)$. Then the clause for the abstraction operator in such a system would presumably look like this:

$$[\![(\lambda x.\phi)(a)]\!]^{c,v,g} = 1 \text{ iff } [\![\phi]\!]^{c,v,g[x/i_a]} = 1$$

And on this sort of approach, the formula $x \neq a$ would presumably be associated with the following truth-conditions:

$$[\![x \neq a]\!]^{c,v,g} = 1 \text{ iff } g(x)(v) \neq i_a(v)$$

This yields the result that sentences like (3) and (7) are true at no point of evaluation. For consider the following:

$$[\![(\lambda x.\Diamond x \neq a)(a)]\!]^{c,v,g} = 1 \text{ iff}$$

$$[\![\Diamond x \neq a]\!]^{c,v,g[x/i_a]} = 1 \text{ iff}$$

$$\text{there is a world } v' \in \mathcal{B}_c \text{ such that } [\![x \neq a]\!]^{c,v',g[x/i_a]} = 1 \text{ iff}$$

$$\text{there is a world } v' \in \mathcal{B}_c \text{ such that } g[x/i_a](x)(v') \neq i_a(v')$$

The crucial point to note here is that $g[x/i_a](x)$ is simply i_a. Thus, the sentence will be true at this point of evaluation just in case:

$$\text{there is a world } v' \in \mathcal{B}_c \text{ such that } i_a(v') \neq i_a(v')$$

But of course there can be no world v' such that $i_a(v')$ is distinct from $i_a(v')$. So sentences like (3) and (7) are predicted to be inconsistent (i.e. false at every point of evaluation) on this approach. Sentence (6), on the other hand, will be true at some points of evaluation since the proposal allows that descriptions like 'the inventor of bitcoin' may be non-rigid over the set of worlds over which epistemic 'might' quantifies.

These are nice results. This approach seems to achieve scopelessness without rigidity: definite descriptions like 'the inventor of bitcoin' may be non-rigid across $\mathcal{B}_c$

[8]See also Bressan 1972, Gibbard 1975, Aloni (2001, 2005), Fitting 2004, Garson (2006), and Holliday and Perry 2014.

despite being scopeless with respect to epistemic 'might.' Unfortunately, there is a problem with this approach.

The problem is that the mechanism by which this approach achieves the scopelessness of descriptions is too general. As a result, it ends up predicting that definite descriptions are scopeless not just with respect to epistemic modals, but with respect to *all* modals. And this means that the theory upends Kripke's pleasing story about the interaction between singular terms and metaphysical modals.

The main issue is that this approach predicts that sentences like (2) are also unambiguously false. For think of the *de re* logical form of (2):

$$(\lambda x.\Diamond_m x \neq p)(p)$$

If the variable x ranges over individuals concepts, then $(\lambda x.\Diamond_m x \neq p)$ is looking for an individual concept. If we then hand it the intension i_p of p, the resulting formula will be true at the actual world v just in case there is a (metaphysically accessible) world v' at which $i_p(v')$ is distinct from the referent of p at v'. But, again, there is no such world, since, for each world v', i_p maps v' to the referent of p at v' – again, that's just how the intension of p is defined. Thus, (2) will be false on its *de re* reading. Since we agreed earlier that it was false on its *de dicto* reading, it will be false on both readings. Thus, the present theory will no longer predicts the contrast between (1) and (2) with which we began, for it will predict that both (1) and (2) are unambiguously false.

The difference between sentences (2) and (3) reveals an asymmetry between how definite descriptions interact with epistemic modals versus how they interact with metaphysical modals. The individual concepts theory fails to respect this asymmetry. We need an approach that posits a deeper semantic difference between epistemic and metaphysical modals.

3 Dynamic semantics

In this section, I make the case that a certain version of dynamic semantics can solve our two puzzles, while at the same time respecting Kripke's observation about sentences (1) and (2).

3.1 The two puzzles

The version of dynamic semantics discussed here is based on the theory of Groenendijk *et al.* (1996), a theory which combines a dynamic theory of epistemic modals with a theory of quantification.[9] The principal differences between that formulation and the one offered here flow from the fact that, unlike Groenendijk et al., we are interested

[9]The theory of Groenendijk *et al.* (1996) is something of a synthesis of the theory of anaphora and quantification found in Groenendijk and Stokhof (1991) and the theory of epistemic modals offered in Veltman (1996).

in a language that contains both an abstraction operator and a metaphysical modal operator. In addition, since we are not concerned with issues surrounding inter-sentential anaphora, we can simplify the approach of Groenendijk et al. in certain respects.

In dynamic semantics, sentences are not assigned truth values relative to a sequence of parameters (at least not if the parameters contain only the usual suspects, such as context, world, variable assignment, etc.). Rather, the meaning of a sentence is understood in terms of its capacity to update a body of information. Meanings are not truth-conditions, but "context-change potentials."

'Context' in the relevant sense means something like 'information being taken for granted' in approximately Stalnaker's sense (Stalnaker 1974, 1978). On the Stalnakerian approach, such information would be represented by a set of possible worlds. But to give a recursive semantics for a language that includes quantifiers and variables, we work with a slightly more technical notion of a state of information. Let's say that a *possibility* is pair of a possible world and a variable assignment, where a variable assignment is a function from variables to individuals. Then a dynamic *state of information* is a set of possibilities in this sense: a state is a set of world-assignment pairs. But we can connect Stalnaker's contexts and the dynamic semanticist's states via the following definition:

Definition 3. Let σ be a set of possible worlds. Then we can say that a set s of possibilities *represents* σ just in case there is a variable assignment g such that $s = \{\langle v, g \rangle : v \in \sigma\}$.

This notion will become important when we try to characterize the notion of *assertability*.

Now atomic formulas, including those containing free variables, can be said to be true or false relative to a possibility $\langle v, g \rangle$ in a straightforward way. The formula $x = a$, for example, will be true at $\langle v, g \rangle$ just in case $v(a) = g(x)$, where $v(a)$ is the referent of a at v. Then we can say that to update a state s with an atomic formula ϕ, we simply throw out of s all the possibilities $\langle v, g \rangle$ at which ϕ is false. Suppose, for example, that we do not know initially whether the inventor of bitcoin is the CEO of Tesla or not. Then if s is a set of possibilities that represents our state of knowledge, there will be possibilities $\langle v, g \rangle$ in s at which $b = o$ is true and possibilities at which $b = o$ is false. If we then learn that the inventor of bitcoin is indeed the CEO of Tesla, we update s with $b = o$ which yields a new state s' which contains all and only those possibilities in s at which $b = o$ is true.

To formalize this a bit, let's adopt the following notation:

Definition 4. Let $i = \langle v, g \rangle$ be a possibility. Then:

If a is an individual constant, $i(a)$ is $v(a)$, the referent of a at v.

If x is a variable, $i(x) = g(x)$.

If P is an n-ary predicate, then $i(P)$ is a set of n-ary sequences of individuals, namely the set of n-ary sequences that satisfy P at v.

If t is a variable or an individual constant, then t is a *term*.

The semantics takes the form of a recursive definition of *the update of a state s with a formula* ϕ, written $s[\phi]$. To see what this approach says about sentences (6) and (8), we need to look at the recursive clauses for idsentity, negation, conjunction, and the epistemic possibility modal.[10]

Definition 5. For any state s:

$$s[t_1 = t_2] = \{i \in s : i(t_1) = i(t_2)\} \quad \text{(where } t_1 \text{ and } t_2 \text{ are terms)}$$

$$s[\neg\phi] = s - s[\phi]$$

$$s[\phi \wedge \psi] = s[\phi][\psi]$$

$$s[\Diamond_e \phi] = \begin{cases} s & \text{if } s[\phi] \neq \emptyset; \\ \emptyset & \text{otherwise.} \end{cases}$$

If ϕ is atomic, then updating s with $s[\neg\phi]$ simply amounts to throwing out all the possibilities in s at which ϕ is true. And if ϕ is atomic or the negation of an atomic formula, we can think of $\Diamond_e\phi$ as "testing" a state of information s for compatibility with ϕ. If s contains a possibility at which ϕ is true, then s passes the test, and the update with $\Diamond_e\phi$ simply returns s. If s contains no such possibility, s fails the test, and $\Diamond_e\phi$ returns the empty set.

In our original description of the data, we described sentences (6) and (8) as true in our context. But dynamic semantics does not usually aim to recover ordinary intuitions about the truth and falsity of sentences.[11] Nevertheless, we can connect the abstract semantics we've been sketching to our puzzle by talking about *assertability* and *inconsistency*. Let's begin with assertability, since that is the notion relevant to (6) and (8).

To give an account of assertability, we first need the following definition:

Definition 6. A state of information s *supports* a formula ϕ just in case $s[\phi] = s$.

The idea then is that a sentence ϕ is assertable for a speaker x just in case x's state of knowledge supports ϕ. Here is the basic idea:

[10] The semantics officially takes the form of a single recursive definition, but for ease of exposition, I split the semantics into separate definitions in what follows (**Definitions 5, 8** and **10**). The full recursive semantics can be found as a single definition in the appendix.

[11] I spoke above about *atomic* formulas and negations of atomics as true or false at a possibility. But it is not clear how to define a suitable notion of truth that applies to formulas in general within this semantics. The problem arises in connection with the epistemic modal; see Stokke (2014) for one attempt to resolve it.

> Sentence ϕ is assertable for agent x in context c iff there is a state s that (a) supports ϕ, and (b) represents the set of possible worlds compatible with what x knows in c.[12]

(See the notion of 'representation' characterized in **Definition 3**.)

Now given the clauses for the epistemic modal, negation, and identity, we have the following result, which is relevant to sentences (6) and (8):

Fact 1.

$$s[\Diamond_e t_1 = t_2] = \begin{cases} s & \textit{if there is an } i \in s \textit{ s.t. } i(t_1) = i(t_2); \\ \emptyset & \textit{otherwise.} \end{cases}$$

$$s[\Diamond_e t_1 \neq t_2] = \begin{cases} s & \textit{if there is an } i \in s \textit{ s.t. } i(t_1) \neq i(t_2); \\ \emptyset & \textit{otherwise.} \end{cases}$$

So let s be a set of possibilities that represents our state of knowledge. Since we don't know whether or not the CEO of Tesla is the inventor of bitcoin, there will be possibilities $i \in s$ such that $i(o) = i(b)$, and possibilities $i' \in s$ such that $i(o) \neq i(b)$. Given **Fact 1**, this means that s supports both $\Diamond_e o = b$ and $\Diamond_e o \neq b$. Given the semantics for conjunction, this means that s supports (6). So dynamic semantics predicts that (6) is assertable in our context.

Now we can say the same thing about sentence (8), but only if we are willing to accept that one of the names "Elon Musk" (m) and "Satoshi Nakomoto" (n) is non-rigid over s. For if they are both rigid over that set, then either $m = n$ is true at every possibility in that set, or $m \neq n$ is true at every possibility in that set. Either way, s will end up failing to support one of the conjuncts of (8). So let's assume for the moment that at least one of those names is non-rigid over s; we'll return to this assumption later in the essay.

Now, our goal is to achieve epistemic scopelessness without epistemic rigidity. How does the dynamic approach help with this project? To answer this, we first need to examine the semantics for the abstraction operator. And to state that proposal, we first need to put some notation in place:

Definition 7. Let $i = \langle v, g \rangle$ be a possibility, a an individual constant, and s a state. Then:

$$i[x/a] = \langle v, g[x/i(a)] \rangle$$

$$s[x/a] = \{i[x/a] : i \in s\}.$$

Note that $g[x/i(a)]$ is the variable assignment that maps x to $i(a)$ and maps each variable y distinct from x to $g(y)$. Then we can state the clause for the abstraction operator as follows:

[12] This is a way of incorporating something like the "knowledge norm" of assertion (Williamson 1996, 2000) into the dynamic framework.

Definition 8. For any state s:

$$s[(\lambda x.\phi)(a)] = \{i \in s : i[x/a] \in s[x/a][\phi]\}$$

Here is what this says. To update a state with a formula of the form $(\lambda x.\phi)(a)$, first take each possibility $\langle v, g\rangle$ in s and alter the associated variable assignment g to a new assignment $g[x/i(a)]$ that is just like g with the possible exception that $g[x/i(a)]$ maps x to the referent of a at i. Then collect up all of these altered possibilities into a new state $s[x/a]$. Now update $s[x/a]$ with ϕ in the usual way. If $\langle v, g[x/i(a)]\rangle$ survives this update, then $\langle v, g\rangle$ survives the update of s with $(\lambda x.\phi)(a)$.

Now this approach does indeed predict that singular terms are scopeless with respect to epistemic modals in the following sense:

Fact 2. *If ϕ does not contain $\Diamond_m$, then for any state s and any individual constant a:*

$$s[(\lambda x.\Diamond_e\phi)(a)] = s[\Diamond_e\phi(a/x)]$$

Thus, on this approach, the *de dicto*-*de re* distinction, as it pertains to epistemic modals and singular terms, essentially collapses. Note that the result holds for all singular terms a, whether rigid or not. So we have epistemic scoplessness without epistemic rigidity.

Let's examine what this means for sentences (3) and (9). We initially characterized our observations about sentences (3) and (9) by saying that they are unambiguously false in our context. In the dynamic setting, we might instead say that both the *de dicto* and the *de re* forms of these sentences are such that their negations are assertable in our context. But actually some stronger holds: these sentences are unambiguously *inconsistent* according to dynamic semantics, where a formula ϕ is *inconsistent* just in case, for any state s, $s[\phi] = \emptyset$.

The *de dicto* forms corresponding to these sentences

$$\Diamond_e a \neq a$$

are obviously inconsistent, given **Fact 1**. For there is no possibility i such that $i(a) \neq i(a)$, whether or not a is rigid. Thus, if we take ϕ to be $x \neq a$, **Fact 2** implies that the corresponding *de re* forms

$$(\lambda x.\Diamond_e x \neq a)(a)$$

are likewise inconsistent. This again holds whether or not a is rigid. Thus, dynamic semantics predicts that sentences like (3) and (9) are unambiguously inconsistent.

The proof of **Fact 2** is sketched in the appendix. But we can get a sense of why it holds by examining the consequence of it that matters most to us: that fact that sentences of the form $(\lambda x.\Diamond_e x \neq a)(a)$ are inconsistent on this semantics.

To see why this holds, let s be any state, and let a be any constant. Note that we make no assumptions about whether or not a is rigid across s. By the clause for the abstraction operator, we have:

$$s[(\lambda x.\Diamond_e x \neq a)(a)] = \{i \in s : i[x/a] \in s[x/a][\Diamond_e x \neq a]\}$$

We want to show that this set – call it s' – is empty. Now, note that, if $s[x/a][\Diamond_e x \neq a]$ is empty, then s' will be empty. And given the clause for the epistemic possibility modal, we have:

$$s[x/a][\Diamond_e x \neq a] = \begin{cases} s[x/a] & \text{if } \; s[x/a][x \neq a] \neq \emptyset; \\ \emptyset & \text{otherwise.} \end{cases}$$

It follows that if $s[x/a][x \neq a]$ is empty, $s[x/a][\Diamond_e x \neq a]$ will be empty, in which case s' will be empty. So it will suffice to show that $s[x/a][x \neq a]$ is empty.

To see this, suppose otherwise. Suppose that there is an $i[x/a]$ in $s[x/a][x \neq a]$.[13] Then, given the clauses for negation and identity, it follows that

$$i[x/a](x) \neq i[x/a](a).$$

Now $i[x/a]$ is, essentially, the possibility that is exactly like i with the possible exception that $i[x/a](x)$ is equal to $i(a)$. So we have:

$$i[x/a](x) = i(a).$$

But since $i[x/a]$ is otherwise like i, it follows that

$$i[x/a](a) = i(a).$$

Thus, it follows that:

$$i[x/a](x) = i[x/a](a).$$

But this contradicts our earlier claim that $i[x/a](x) \neq i[x/a](a)$. So $s[x/a][x \neq a]$ must be empty after all, which is what we needed to show.

The crucial point is that, when we assess $s[x/a][\Diamond_e x \neq a]$, $\Diamond_e$ is looking for a possibility in $s[x/a]$ at which $x \neq a$ holds. If it finds one, it returns $s[x/a]$; if it fails to find one, it returns the empty set. Now, each possibility in $s[x/a]$ is of the form $i[x/a]$ (for some $i \in s$). And as we just saw, x and a have the same referent at any such possibility: both pick out the referent of a at i. Thus, $x \neq a$ will fail at any such possibility. Note that this holds even if both x and a are non-rigid across $s[x/a]$. For even if they are non-rigid across $s[x/a]$, they will *coincide in reference with each other* at each possibility in $s[x/a]$. And that is what matters.

[13] By the clause for negation, every possibility in $s[x/a][x \neq a]$ is in $s[x/a]$, and every possibility i' in $s[x/a]$ is such that there is an i in s such that $i' = i[x/a]$.

3.2 Descriptions and metaphysical modality

So dynamic semantics solves our two puzzles. But so far we've only reproduced the results of the individual concepts theory of §2 using somewhat more elaborate machinery. We need to reassure ourselves that the theory preserves Kripke's original observations about sentences (1) and (2). This turns out to be more straightforward in the case of sentence (2) – the case of descriptions – so we deal with that case first before turning to sentence (1) and the case of proper names.

To see what the dynamic theory says about sentence (2), we need to see what it says about the metaphysical possibility modal. To state the clause for this modal, we appeal to a binary accessibility relation $\mathcal{R}$ across the space of possible worlds. Since this is intended to model 'metaphysical accessibility,' we may assume for the moment that $\mathcal{R}$ is universal, that is, that for any $v, v' \in \mathcal{W}$, $v\mathcal{R}v'$. But we will revisit this assumption later when we discuss names.

To state the clause for the metaphysical possibility modal, we employ the following notation:

Definition 9. Let $i = \langle v, g \rangle$ be a possibility. Then:

$$\mathcal{R}(v) = \{v' : v\mathcal{R}v'\}$$

$$\mathcal{R}(i) = \{\langle v', g' \rangle : g' = g \;\&\; v' \in R(v)\}$$

Then we have the following clause for the metaphysical modal:

Definition 10. For any state s:

$$s[\Diamond_m \phi] = \{i \in s : \mathcal{R}(i)[\phi] \neq \emptyset\}$$

If ϕ is an atomic or the negation thereof, then to update a state s with $\Diamond_m \phi$, we simply throw out all possibilities i in s that cannot 'metaphysically access' a world at which ϕ is true.

What we want to show is that dynamic semantics predicts that the *de re* form of (2)

$$(\lambda x.\Diamond_m x \neq p)(p)$$

is assertable for us. Thus, we want to show that if s is a set of possibilities that represents the set of possible worlds compatible with what we know, then:

$$s[(\lambda x.\Diamond_m x \neq p)(p)] = s.$$

Since the corresponding *de dicto* formula is inconsistent according to dynamic semantics, this will mean that ordinary definite descriptions are not scopeless with respect to metaphysical modals. There will be a substantive *de re*-*de dicto* distinction for descriptions and metaphysical modals.

Now we assume that definite descriptions like 'the President of the U.S. in 2018' are metaphysically non-rigid. In the context of dynamic semantics, this means that, given any state of information s that we might occupy, if $\langle v, g\rangle$ is an element of s, then 'the President of the U.S. in 2018' is non-rigid across $\mathcal{R}(v)$. So it will suffice to establish the following result:

Fact 3. *Let s be a state such that, for each possibility $\langle v, g\rangle$ in s, a is non-rigid across $\mathcal{R}(v)$. Then:*

$$s[(\lambda x.\Diamond_m x \neq a)(a)] = s.$$

To see why this holds, let s be a state such that, for each $\langle v, g\rangle$ in s, a is non-rigid over $\mathcal{R}(v)$. From the clause for the abstraction operator, we have:

$$\begin{aligned} & s[(\lambda x.\Diamond_m x \neq a)(a)] \\ & = \{i \in s : i[x/a] \in s[x/a][\Diamond_m x \neq a]\} \\ & = \{\langle v, g\rangle \in s : \langle v, g'\rangle \in s[x/a][\Diamond_m x \neq a]\}, \text{ where } g' = g[x/v(a)] \end{aligned}$$

Let $\langle v, g\rangle$ be any possibility in s, and let $g' = g[x/v(a)]$. To show that

$$s[(\lambda x.\Diamond_m x \neq a)(a)] = s,$$

it will suffice to show that

$$\langle v, g'\rangle \in s[x/a][\Diamond_m x \neq a]$$

Given the clause for the metaphysical modal operator, this holds just in case:

$$\mathcal{R}(v, g')[x \neq a] \neq \emptyset$$

Note that:

$$\mathcal{R}(v, g') = \{\langle v', g''\rangle : g'' = g' \;\&\; v' \in \mathcal{R}(v)\}.$$

So, given the clauses for negation and identity, we need to show that there is a v' in $\mathcal{R}(v)$ such that $g'(x) \neq v'(a)$. Note that $g'(x) = v(a)$, since $g' = g[x/v(a)]$ and $g[x/v(a)](x) = v(a)$. So we want to show that there is a v' in $\mathcal{R}(v)$ such that $v(a) \neq v'(a)$. Suppose, for *reductio*, that there were no such world. Then a would be rigid across $\mathcal{R}(v)$. But that contradicts our assumption that a is non-rigid across $\mathcal{R}(v)$. So there must be such a v' in $\mathcal{R}(v)$, which is what we needed to show.

The crucial difference between the epistemic possibility modal and the metaphysical possibility modal on this approach is this. Once the abstraction operator has done its work, $\Diamond_e$ is essentially acting as a quantifier over $s[x/a]$. And, as we noted above, for each possibility $i[x/a]$ in $s[x/a]$, the extension of x at $i[x/a]$ is identical to the extension of a at $i[x/a]$. Hence, there is no relevant possibility at which $x \neq a$ holds.

In contrast, once the abstraction operator has done its work in the metaphysical case, $\Diamond_m$ quantifies over $\mathcal{R}(v, g')$, for each $\langle v, g' \rangle$ in $s[x/a]$. By assumption, a is non-rigid over $\mathcal{R}(v)$, which means that it is non-rigid over $\mathcal{R}(v, g')$. But x, on the other hand, is *rigid* over $\mathcal{R}(v, g')$.[14] So since a is non-rigid over the relevant set of possibilities, and x is rigid over that same set, they must come apart in extension at some relevant possibility. Thus, $x \neq a$ will hold at some relevant possibility.

3.3 Names and metaphysical modality

So the dynamic theory improves on the individual concepts theory. Both approaches solve our initial two puzzles (at least given the assumption that names are epistemically non-rigid), but the individual concepts approach runs into trouble with sentence (2), incorrectly predicting that it has no acceptable reading. The dynamic approach, on the other hand, predicts that that sentence is assertable in our context on its *de re* reading.

But there is one outstanding issue: the status of sentence (1). We began the essay with Kripke's contrast between (1), which has no acceptable reading, and (2), which does. Dynamic semantics predicts the latter; can it also predict the former?

There is a *prima facie* difficulty. To see the problem, recall that we have been assuming that $\mathcal{R}$ is the *universal relation* over the space of possible worlds, so that $v\mathcal{R}v'$ holds for any worlds v, v'. This is a natural (though not inevitable) assumption, given that $\mathcal{R}$ is the metaphysical accessibility relation. And note that (8) will be assertable in our context only if one of the two names that figures in that sentence is non-rigid over the space $\mathcal{W}$ of possible worlds, at least if the states that figure in dynamic semantics employ worlds drawn from $\mathcal{W}$.[15]

But if $\mathcal{R}$ is universal, and if one of the names in (8) is non-rigid over $\mathcal{W}$, then either every state will support the *de re* form of (11) or every state will support the *de re* form of (12):

(11) Elon Musk might not have been Elon Musk.

(12) Satoshi Nakomoto might not have been Satoshi Nakomoto.

[14] This is because, for each possibility $\langle v', g'' \rangle \in \mathcal{R}(v, g')$, $g'' = g'$. So any pair of possibilities i, i' in $\mathcal{R}(v, g')$ will share the same variable assignment, which means that x will not vary in extension over i, i'.

[15] To see this, let s be any state, and suppose that both names, m and n, are rigid over $\mathcal{W}$. We show that $s[(8)] = \emptyset$. Since m and n are rigid, it is either the case that, for each i in s, $i(m) = i(n)$, or, for each i in s, $i(m) \neq i(n)$. And whichever possibility obtains, $s[(8)]= \emptyset$. To see this, first suppose that for every i in s, $i(m) \neq i(n)$. Then $s[m = n] = \emptyset$, which means that $s[\Diamond_e m = n] = \emptyset$. And if $s[\Diamond_e m = n] = \emptyset$, then $s[(8)]= \emptyset$. Now suppose that for every i in s, $i(m) = i(n)$. In that case, $s[\Diamond_e m = n] = s$, since $s[m = n] = s$. But since $s[m = n] = s$, $s - s[m = n] = \emptyset$, which means that $s[m \neq n] = \emptyset$. And this, in turn, means that $s[\Diamond_e m \neq n] = \emptyset$. But if $s[\Diamond_e m \neq n] = \emptyset$, then $s[(8)]= \emptyset$.

To see this, suppose, for example, that 'Elon Musk' is non-rigid over $\mathcal{W}$ and that $\mathcal{R}$ is universal. Then if s is any state, and $\langle v, g\rangle$ any possibility in s, then $\mathcal{R}(v)$ is simply the set of possible worlds – this follows from $\mathcal{R}$'s universality. Thus, since 'Elon Musk' is (by supposition) non-rigid over the set of possible worlds, it is non-rigid over $\mathcal{R}(v)$. **Fact 3** then tells us that s supports the *de re* form of (11).

But sentences (11) and (12) are just like sentence (1), and we agreed with Kripke that such sentences are unambiguously unacceptable. So the problem is that, given the above two assumptions ($\mathcal{R}$ is universal, not all proper names are rigid over $\mathcal{W}$), dynamic semantics would seem to be committed to some sentence of this form being assertable (on its *de re* reading) in any context.

There are at least three possible responses to this problem.

One response is to accept that names are rigid over the space of possible worlds, and accept that the $\langle v, g\rangle$ possibilities that figure in dynamic semantics contain genuine possible worlds. Then sentences like (1) will be predicted to be inconsistent.[16] (This is the sort of approach a "Millian" about names might endorse.) If we took this approach, we would then need to explain the apparent acceptability of sentences like (8) in some other way. This avoids the problem, but it remains to be seen how the the apparent acceptability of (8) will be explained.

A second response would be to accept that names are not rigid across the whole of $\mathcal{W}$, and to deny the universality of $\mathcal{R}$. From the point of view of certain worlds in $\mathcal{W}$, certain other worlds in $\mathcal{W}$ are not metaphysically accessible. How would this help? Well, if names need not be rigid across the whole of $\mathcal{W}$, then we could have a state s that contains possibilities i, i' such that $i(m) = i(n)$ and $i'(m) \neq i'(n)$. Such a state would support (8), and thus predict the potential assertability of that sentence.

But how would this approach predict the inconsistency of sentences like (1)? Well, suppose we adopted the following constraint:

> For each possible world v, and each name a, a is rigid over $\mathcal{R}(v)$.

If this constraint is adopted, then sentences like (1) will turn out to be inconsistent.[17] The idea here is that, while names are not rigid across the whole of $\mathcal{W}$, they are "locally rigid," i.e. rigid across each set of metaphysically accessible worlds $\mathcal{R}(v)$, for each world v.[18]

[16] See footnote 17. (Note that if a name n is rigid over $\mathcal{W}$, it will be rigid over $\mathcal{R}(v)$, for any $v \in \mathcal{W}$, as long as $\mathcal{R}$ is a subset of $\mathcal{W} \times \mathcal{W}$.)

[17] To see this, let a be an individual constant that is rigid over $\mathcal{R}(v)$, for each possible world v. Let s be any state. Suppose, for *reductio*, that there is a possibility $\langle v, g\rangle$ in

$$s[(\lambda x.\Diamond_m x \neq a)(a)].$$

Then by the clause for the abstraction operator, $\langle v, g[x/v(a)]\rangle \in s[x/a][\Diamond_m x \neq a]$. So by the clause for the metaphysical modal, $\mathcal{R}(v, g[x/v(a)][x \neq a] \neq \emptyset$. So there is a $v' \in \mathcal{R}(v)$ such that $g[x/v(a)](x) \neq v'(a)$. Since $g[x/v(a)](x) = v(a)$, it follows that $v(a) \neq v'(a)$. But since v and v' are in $\mathcal{R}(v)$ (note that $\mathcal{R}$ is reflexive), and since a is rigid across $\mathcal{R}(v)$, it follows that $v(a) = v'(a)$. Contradiction.

[18] See Fitting and Mendelsohn (1998, §10.2) for further discussion.

Note a feature of this approach. Suppose that s represents our state of information, that $\langle v, g\rangle$ is a possibility in s such that $v(m) = v(n)$, and that $\langle w, g\rangle$ is a possibility in s such that $w(m) \neq w(n)$. On the present proposal, every world v' metaphysically accessible from v will be such that $v'(m) = v'(n)$. So that means that w is not metaphysically accessible from v.

How should we interpret this situation? The precise description of the situation is somewhat delicate, given the dynamic setting in which we are operating. But it is clear that this picture has affinities with the idea that there are epistemic possibilities that are not metaphysically possible (Soames 2009). For it seems that, from the point of view of v, w is epistemically, but not metaphysically, possible.

Now, the idea that there are epistemic possibilities that are not metaphysically possible might sound like a mere truism to some, especially if one considers mathematical or logical falsehoods which we do not know to be false. I am sympathetic to this view. But when it comes to the sorts of issues we are dealing with – which involve empirical ignorance of identity claims – there is a tradition of thinking that we can handle these problems without bifurcating modal space in this way. Readers sympathetic to *two-dimensional semantics* may thus prefer a third solution to the present difficulty, which amounts to adapting the two-dimensional approach into the dynamic setting. One of the guiding ideas behind two-dimensionalism is that we have, not two modal spaces, but two ways of evaluating expressions and sentences with respect to a world.[19] Two-dimensional idea can be implemented with a dynamic framework, and the resulting theory also resolves our puzzle about names. Considerations of space prevent from justifying this claim here, but I hope to pursue it in future work.

So we have three potential solutions to the problem of reconciling the assertability of (8) with the inconsistency of sentences like (11) and (12). Since only the second potential solution was sketched in any detail, we leave it open which of these options is to be preferred. The point to observe here is that this problem does not sink dynamic semantics, for the dynamic semanticist has various things she can say in response to it.

4 Conclusion

The version of dynamics semantics developed here accounts for the interaction between modals (epistemic and metaphysical) and singular terms (names and descriptions). But although I have in effect argued against two "static" theories of these matters (in §§1-2), I have not argued that no static theory of these matters can be given – I have not argued that these puzzles demonstrate a *need* for dynamic semantics.[20] A more thorough investigation would undertake that question; here, we simply

[19]For the sorts of views I have in mind, see Davies and Humberstone (1980), Jackson (1998), and Chalmers (2006a,b) among others.

[20]For static approaches that might account for our data, see Mandelkern (2017, 2018), Rabern (2018), and AUTHOR (XX).

close by mentioning two considerations that might be taken to bear on it.

First, we have focused on one class of *de re-de dicto* distinctions, namely those involving singular terms. But of course the interaction of quantifiers and epistemic modals also give rise to *de re-de dicto* ambiguities, and so any full theory of these matters will have to encompass a theory of quantified epistemic modality.[21] Second, although we have been discussing both epistemic and metaphysical modals, we have not discussed their interaction in any detail. What happens when a metaphysical modal occurs within the scope of an epistemic modal, or vice-versa? This question may also bear on the choice between static and dynamic semantics.

In any case, these questions are left as ones for future inquiry.

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[21] Quantified epistemic modality is discussed in Groenendijk *et al.* 1996, Aloni (2001) Beaver 2001, Yalcin 2015, Klinedinst and Rothschild 2016, Mandelkern 2017, 2018, Moss (2018), Rabern (2018), and AUTHOR (XX).

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Appendix

In order to state the full recursive semantics discussed in §3, we need another piece of notation:

Definition 11. Let $i = \langle v, g\rangle$ be a possibility, and o an object. Then:

$$i[x/o] = \langle v, g[x/o]\rangle$$

$$s[x/o] = \{i[x/o] : i \in s\}$$

Then we have the following:

Definition 12. Let s be any state. *The update of s by ϕ, $s[\phi]$*, is defined as follows:

$$s[P(t_1, ..., t_n)] = \{i \in s : \langle i(t_1), ..., i(t_n)\rangle \in i(P)\}$$

$$s[t_1 = t_2] = \{i \in s : i(t_1) = i(t_2)\}$$

$$s[\neg\phi] = s - s[\phi]$$

$$s[\phi \wedge \psi] = s[\phi][\psi]$$

$$s[(\lambda x.\phi)(a)] = \{i \in s : i[x/a] \in s[x/a][\phi]\}$$

$$s[\exists x\phi] = \{i \in s : \text{there is an object } o \text{ s.t. } i[x/o] \in s[x/o][\phi]\}$$

$$s[\Diamond_m\phi] = \{i \in s : \mathcal{R}(i)[\phi] \neq \emptyset\}$$

$$s[\Diamond_e\phi] = \begin{cases} s & \text{if } s[\phi] \neq \emptyset; \\ \emptyset & \text{otherwise.} \end{cases}$$

The other logical expresions may be understood as abbreviations in the usual way, e.g. $(\phi \vee \psi)$ is $\neg(\neg\phi \wedge \neg\psi)$, $\Box_m\phi$ is $\neg\Diamond_m\neg\phi$, etc..

Recall **Fact 2**, which says:

> If ϕ does not contain $\Diamond_m$, then for any state s and any individual constant a:
>
> $$s[(\lambda x.\Diamond_e\phi)(a)] = s[\Diamond_e\phi(a/x)]$$

This is in effect what it means to say (in the context of dynamic semantics) that a singular term is scopeless with respect to the epistemic modal. This claim follows from the following more general claim, the proof of which is sketched here:

Fact 4. *If ϕ does not contain $\Diamond_m$, then for any state s and any individual constant a:*

$$s[(\lambda x.\phi)(a)] = s[\phi(a/x)]$$[22]

Proof. The proof is by induction on the complexity of formulas, where the complexity of a formula is the number of logical symbols it contains (excluding identity). Here we discuss the base case and the cases for negation and the epistemic modal. Note that these are the cases most relevant to the formulas which we have been discussing.

Base case. Let $P(t_1, ..., t_n)$ be an arbitrary atomic formula, let s be any state. We want to show:

$$s[(\lambda x.P(t_1, ..., t_n))(a)] = s[P(t_1, ..., t_n)(a/x)]$$

Let's use the following notation: for any of the t_i ($1 \leq i \leq n$):

$$t_i(a/x) = \begin{cases} a & \text{if } t_i \text{ is the variable } x; \\ t_i & \text{otherwise.} \end{cases}$$

So $P(t_1, ..., t_n)(a/x)$ is $P(t_1(a/x), ..., t_n(a/x))$. So we need to show:

$$s[(\lambda x.P(t_1, ..., t_n))(a)] = s[P(t_1(a/x), ..., t_n(a/x))]$$

Let i be any possibility. From the clauses for the abstraction operator and atomic formulas we have:

$$i \in s[(\lambda x.P(t_1, ..., t_n)(a)] \text{ iff}$$
$$i[x/a] \in s[x/a][P(t_1, ..., t_n)] \text{ iff}$$
$$\langle i[x/a](t_1), ..., i[x/a](t_n)\rangle \in i[x/a](P) \text{ iff}$$
$$\langle i[x/a](t_1), ..., i[x/a](t_n)\rangle \in i(P)$$

And from the clause for atomic formulas, we have:

$$i \in s[P(t_1(a/x), ..., t_n(a/x))] \text{ iff}$$
$$\langle i(t_1(a/x)), ..., i(t_n(a/x))\rangle \in i(P)$$

So it will suffice to show that, for each of the t_i, $i[x/a](t_i) = i(t_i(a/x))$. So let t be any of the t_i. There are two cases: either t is x or t is not x.

First suppose that t is x. In that case $i[x/a](t) = i[x/a](x) = i(a)$. And since t is x, $t(a/x)$ is a. So $i(t(a/x)) = i(a)$. It follows that $i[x/a](t) = i(t(a/x))$.

Now suppose that t is not x. In that case, $i[x/a](t) = i(t)$. And since t is not x, $t(a/x)$ is t. So $i(t(a/x)) = i(t)$. So it again follows that $i[x/a](t) = i(t(a/x))$.

[22] As noted earlier, $\phi(a/x)$ is the result of replacing all free occurrences of x with a. The free occurrences of x in an atomic formula ϕ are all occurrences of x in ϕ. The free occurrences of x in a complex formula ϕ are those of its principal subformulas with the following exception: if ϕ is of the form $\exists x\psi$ or $(\lambda x.\psi)(a)$, then there are no free occurrences of x in ϕ.

The case of formulas of the form $t_1 = t_2$ is simply the case where P is $=$, and $t_1, ..., t_n$ is t_1, t_2.

Induction step. Let n be a number greater than 0. Our induction hypothesis is that the result holds for every formula with complexity less than n. We want to show that the result holds for every formula ϕ with complexity n.

Negation. Suppose ϕ is $\neg\psi$ for some formula ψ that does not contain $\Diamond_m$. Let s be any state. We want to show:

$$s[(\lambda x.\neg\psi)(a)] = s[(\neg\psi)(a/x)].$$

Since the free variable occurrences in $\neg\psi$ are just those that occur in ψ, $(\neg\psi)(a/x)$ is $\neg\psi(a/x)$. So we want to show:

$$s[(\lambda x.\neg\psi)(a)] = s[\neg\psi(a/x)].$$

By the clause for negation, the induction hypothesis (IH), and the clause for the abstraction operator, we have:

$$\begin{aligned}(13)\quad & s[\neg\psi(a/x)] \\ & = s - s[\psi(a/x)] \\ & = s - s[(\lambda x.\psi)(a)] \quad \text{(by IH)} \\ & = s - \{i \in s : i[x/a] \in s[x/a][\psi]\} \\ & = \{i \in s : i[x/a] \notin s[x/a][\psi]\}.\end{aligned}$$

Now, by the clauses for the abstraction operator and negation and (13), we have

$$\begin{aligned}& s[(\lambda x.\neg\psi)(a)] \\ & = \{i \in s : i[x/a] \in s[x/a][\neg\psi]\} \\ & = \{i \in s : i[x/a] \in (s[x/a] - s[x/a][\psi])\} \\ & = \{i \in s : i[x/a] \in s[x/a] \text{ and } i[x/a] \notin s[x/a][\psi]\} \\ & = \{i \in s : i[x/a] \notin s[x/a][\psi]\}^{23} \\ & = s[\neg\psi(a/x)] \quad \text{(by (13))}\end{aligned}$$

which is what we needed to show.

Epistemic possibility modal. Suppose now that ϕ is $\Diamond_e\psi$, for some formula ψ that does not contain $\Diamond_m$. Let s be any state. We want to show:

$$s[(\lambda x.\Diamond_e\psi)(a)] = s[(\Diamond_e\psi)(a/x)].$$

[23] Note that $i \in s$ iff $i[x/a] \in s[x/a]$.

Note that $(\Diamond_e\psi)(a/x)$ is $\Diamond_e\psi(a/x)$. So we want to show:

$$s[(\lambda x.\Diamond_e\psi)(a)] = s[\Diamond_e\psi(a/x)].$$

Note that if s is empty, then the expression on the left-hand side of the identity sign denotes the empty set, as does the expression on the right-hand side of the identity sign. This is because result of updating the empty set with any formula is itself the empty set. So let us suppose now that s is not empty.

Given the clause for the abstraction operator, it will suffice to show that, for any $i \in s$:

$$i[x/a] \in s[x/a][\Diamond_e\psi] \text{ iff } i \in s[\Diamond_e\psi(a/x)]$$

Give the clause for the epistemic modal, this will hold just in case:

$$s[x/a][\psi] \neq \emptyset \text{ iff } s[\psi(a/x)] \neq \emptyset$$

So we need to show:

There is a j in s s.t. $j[x/a] \in s[x/a][\psi]$ iff there is a k in $s[\psi(a/x)]$

Suppose first that there is a j in s such that $j[x/a] \in s[x/a][\psi]$. Then by the clause for the abstraction operator:

$$j \in s[(\lambda x.\psi(a)].$$

So by the induction hypothesis,

$$j \in s[\psi(a/x)].$$

So there is a k in $s[\psi(a/x)]$, for j is such a k. Now suppose there is a k in $s[\psi(a/x)]$. Then k is in s,[24] and, by the induction hypothesis:

$$k \in s[(\lambda x.\psi(a)]$$

So by the clause for the abstraction operator:

$$k[x/a] \in s[x/a][\psi]$$

So there is a j in s such that $j[x/a] \in s[x/a][\psi]$, for k is such a j. □

[24] Here we rely on the fact that our semantics has the *update property*, i.e. that $s[\chi] \subseteq s$, for all formulas χ.