NAMING AND EPISTEMIC NECESSITY*

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Abstract

Kripke (1980) hypothesizes a link between rigidity and scope: a singular term is rigid over a space S of possibilities just in case it is scopeless with respect to modals that quantify over S. Kripke's hypothesis works well when we consider the interaction of singular terms with metaphysical modals, but runs into trouble when we consider the interaction of singular terms with epistemic modals. After describing the trouble in detail, and considering one non-solution to it, I develop a novel version of dynamic semantics that resolves the problem.

1 Introduction

One of the central theses of *Naming and Necessity* is that proper names are rigid designators, while (most ordinary) definite descriptions are not. Let us remind ourselves of one of Kripke's arguments for this two-fold claim (Kripke, 1980, 48–49).

Consider the contrast between the following pair of sentences:

(1) Trump might not have been Trump.

(2) The President of the U.S. in 2019 might not have been the President of the U.S. in 2019.

The modal here is to be understood as expressing metaphysical possibility. The first of these appears to be unambiguously false, for it seems that no one other than Trump could have been Trump. But things are different with the second sentence. For while it does have a false reading, it also appears to have a true reading. The latter can be brought out by saying,

> 'The person who is in fact the President in 2019—that man—*he* might not have been the President in 2019.'

*Penultimate version, August 2019. Final version forthcoming in *Noûs*.

The parallel speech with 'Trump' replacing 'the President in 2019' is harder to make sense of.

The above contrast between (1) and (2) is elegantly explained by Kripke's hypothesis that proper names are rigid designators while (most ordinary) definite descriptions are not. Let us agree to call both proper name and definite description 'singular terms.' If we ignore the possibility that a singular term might fail to refer at some possible worlds, then we can say:

> A singular term a is *rigid* just in case: for any pair of possible worlds v, v', the referent of a at v is identical to the referent of a at v'.

The explanation of the contrast between (1) and (2) then proceeds as follows. Let us suppose that each of the above sentences has two different "logical forms," a *de dicto* form and a *de re* form. The *de dicto* forms of (1) and (2) correspond (respectively) to the following English sentences:

> It might have been that [Trump is not Trump].
>
> It might have been that [the President of the U.S. in 2019 is not the President of the U.S. in 2019].

These *de dicto* forms may (respectively) be formalized as follows (using t to translate 'Trump' and p to translate 'the President of the U.S. in 2019'):

$$\Diamond_m t \neq t$$

$$\Diamond_m p \neq p$$

(The subscript 'm' here indicates that the modality in question is metaphysical.) These *de dicto* logical forms are both false. For whether or not a is a rigid designator, there is no possible world at which 'a is not a' is true.

So the contrast between (1) and (2) must emerge when we consider their *de re* forms. The *de re* form of (1) corresponds to the following English sentence:

> Trump is such that it might have been that [he was not Trump]

and may be formalized with the help of an "abstraction operator" as follows:

$$(\lambda x.\Diamond_m x \neq t)(t).$$

This logical form is evaluated for truth by first finding the referent r of 'Trump' at the actual world v and then asking whether there is a possible world v' at which r is distinct from the referent r' of 'Trump' at v'. The sentence is true just in case there is such a world. On (and only on) the assumption that proper names like 'Trump' are rigid, there is no such world. For if 'Trump' is rigid, then the referent of 'Trump' at any other world is that very same man r. So there will be no possible world v' at which the referent of 'Trump' at v' is distinct from the actual referent of 'Trump'—for there is no possible world at which r is distinct from r. So the *de re* form of (1) is false if 'Trump' is rigid.

The *de re* form of (2), on the other hand, is predicted to be *true* given the assumption that the description 'the President of the U.S. in 2019' is non-rigid. The *de re* form of this sentence corresponds to the following English sentence:

> The President of the U.S. in 2019 is such that it might have been that [he was not the President of the U.S. in 2019]

and may be formalized as follows:

$$(\lambda x.\Diamond_m x \neq p)(p).$$

Again, we evaluate this form by first finding the referent r of 'the President of the U.S. in 2019' in the actual world v and then asking whether there is a possible world v' in which r is distinct from what 'the President of the U.S. in 2019' refers to in v'. If that description is non-rigid, then there *must* be such a world v'. For if 'the President of the US in 2019' is non-rigid, then there must be a world v' at which it refers not to its actual referent (Trump) but to some other individual (Hillary Clinton, say). For if there were no such world, then 'the President of the U.S. in 2019' would refer to its actual referent in every world, and so would be rigid.

Thus, if the proper name 'Trump' is rigid, both the *de dicto* and *de re* forms of (1) are false. And if the description 'the President of the U.S. in 2019' is non-rigid, then the *de re* form of (2) is true. So this account predicts the apparent asymmetry between (1) and (2).

Some may wish to take issue with some aspect of the foregoing argument; I do not. Instead, what I want to point out is that this sort of reasoning leads us into a puzzle when we move from considering metaphysical modals (Kripke's concern) to *epistemic modals*, the topic of much recent discussion in the philosophy of language.[1]

Let me begin by stating a generalization suggested by the foregoing considerations. Let's say that a singular term is *scopeless* with respect to a modal operator M just in case it doesn't matter whether you interpret it inside or outside the scope of M. More precisely, we can say that:

> A singular term a is *scopeless with respect to modal M* iff for all formulas ϕ, $(\lambda x.M\phi)(a)$ is equivalent to $M\phi(a/x)$, where $\phi(a/x)$ is the result of replacing all free occurrences of x in ϕ with a.

So to say that a singular term a is scopeless with respect to M is to say that a *de re* modal predication featuring a and M is equivalent to its *de dicto* counterpart: the *de re*–*de dicto* distinction, as it pertains to a and M, collapses.

Kripke's argument seems to suggest the following:

> SCOPELESSNESS ⇔ RIGIDITY
>
> A singular term a is scopeless with respect to a modal operator M just in case a is rigid over the domain over which M quantifies.

Proper names are scopeless with respect to metaphysical modals, a fact seemingly explained by their rigidity over the space of metaphysically possible worlds.

[1]Kripke is explicit in various plans that epistemic modals of the sort we're discussing fall outside the scope of his inquiry. See, for example, Kripke (1980, 103, 141, 143).

Ordinary definite descriptions are *not* scopeless with respect to such modals, a fact seemingly explained by their metaphysical *non*-rigidity.[2]

Note that I have stated the above generalization not using the notion of "rigidity full-stop," as it were, but with a more generalized notion of "rigid over" a set of worlds. If we continue to ignore the possibility that a singular term might fail to refer at some worlds, this notion can be defined as follows:

> A singular term a is *rigid over a set S of worlds* just in case, for any worlds v, v' in S, the referent of a in v is identical to the referent of a in v'.

The notion of rigidity Kripke had in mind can then be recovered as follows: a singular term is rigid in Kripke's sense just in case it is rigid over the set of metaphysically possible worlds.

To see what issue epistemic modals raise in this context, consider the epistemic counterpart of sentence (2):

(3) The inventor of bitcoin might not be the inventor of bitcoin.

Unlike sentence (2), it is difficult to hear a true reading of sentence (3), as Aloni (2001, Ch. 3) and Yalcin (2015) observe. Obviously, the *de dicto* form of this sentence will be false. But even the *de re* form seems to be false. To see this, consider the inventor of bitcoin, whoever that may be. Is it true that he or she might not be the inventor of bitcoin? How could that be? That person is, after all, given to us *as* the inventor of bitcoin.

I'm inclined to agree with Aloni and Yalcin that sentences like (3) are unambiguously unacceptable. But if you feel uncertain about this, it might help to consider one of the following instead:

(4) The inventor of bitcoin might turn out not to be the inventor of bitcoin.

(5) We might discover that the inventor of bitcoin is not the inventor of bitcoin.

It's hard for me to hear a true reading of either of these. Again, any reasonable theory will predict that the *de dicto* reading of (5), for example, is false. So focus on the *de re* reading. And consider the inventor of bitcoin, whoever that may be. Might we discover that that person is not, in fact, the inventor of bitcoin? How could we? Whatever course our inquiry takes, we can be sure that it will not lead us to the discovery that that person is not the inventor of bitcoin.

Now, when we move from metaphysical to epistemic modals, we have to deal with the fact that the truth or acceptability of a sentence containing an epistemic modal is not absolute. Precisely how to understand this fact has been the subject of much recent dispute between *contextualists* and *relativists*, but

[2]Kripke makes the connection between scopelessness and rigidity in various places; see, for example, (Kripke, 1980, 12). For a treatment of rigidity and scope in modal logic, see Fitting and Mendelsohn (1998, §10.2).

I don't think those controversies have much bearing on the present issue, for the following reason.[3] It is reasonable for us to proceed by framing the issue in terms of whether or not our target sentences are *assertable* in our present context. In our context c, sentences (3)–(5), for example, are not assertable. But the disputes between contextualists and relativists tend not to concern the assertability of sentences containing epistemic modals, but the conditions under which they may be appropriately disputed or retracted. And as MacFarlane (2014, Ch. 10) argues, contextualists and relativists can agree that assertability is to be evaluated in terms of the notion of "truth at a context." So we should be safe in appealing to that notion, at least in our initial formulation of these issues.

Now we are assuming that (3) has no true reading in our context c. Any reasonable theory will predict that the *de dicto* form of (3) has no true reading in our context. But consider the *de re* form of (3):

$$(\lambda x.\Diamond_e x \neq b)(b)$$

(Here, b translates 'the inventor of bitcoin,' and the subcript 'e' indicates that the modal is epistemic.) The falsity of this form in our context suggests that the definite description 'the inventor of bitcoin' is scopeless with respect to the the epistemic modal 'might.' Thus, if we extend Kripke's line of reasoning—embodied in the principle SCOPELESSNESS $\Leftrightarrow$ RIGIDITY—we are led to the conclusion that that description is rigid over $\mathcal{B}_c$, the domain over which epistemic 'might' quantifies in c. For suppose it were not rigid over $\mathcal{B}_c$. Then if r is the referent of 'the inventor of bitcoin' in the actual world v, there would have to a world v' in $\mathcal{B}_c$ at which that description referred to someone other than r. But such a world would then underwrite the truth of (3) at our context.

So it would seem that, while being *metaphysically non-rigid*, ordinary definite descriptions are nevertheless *epistemically rigid in* c—rigid over the set of worlds over which epistemic 'might' quantifies in c. This is just Kripke's argument from scopelessness to rigidity, adapted to the epistemic case.

But this leads to a problem. For isn't it obvious that the definite description 'the inventor of bitcoin' is epistemically non-rigid in our present context? Here is a fact about bitcoin: no one knows who invented it.[4] But technology journalists and other interested parties have a few names on their list of suspects. One such suspect is Elon Musk, CEO of Tesla; another is an Australian computer scientist named 'Craig Wright.' So it would seem that, in some worlds compatible with what we know, 'the inventor of bitcoin' refers to Elon Musk; in others, to Craig Wright. Thus, it seems that the definite description 'the inventor of bitcoin' is *non-rigid* over the set of worlds $\mathcal{B}_c$ over which 'might' quantifies in our present context.

[3]For discussion of this debate, see Egan et al. 2005, Egan 2007, Stephenson 2007, Yalcin 2007, 2011, von Fintel and Gillies 2008, 2011, Dowell 2011, MacFarlane 2011, 2014, and Schaffer 2011, among others. Related issues are discussed in Hacking 1967 and DeRose 1991.

[4]Well, *someone* (e.g. the inventor) presumably knows, but his or her identity is not (as of mid-2019) a matter of public knowledge. For simplicity, we shall assume that bitcoin was invented by one person, not a committee.

The claim that 'the inventor of bitcoin' is non-rigid over $\mathcal{B}_c$ is also supported by the fact that the following sentence appears to be true in our context:

(6) The CEO of Tesla might be the inventor of bitcoin, but (then again) the CEO of Tesla might not be the inventor of bitcoin.

$$\Diamond_e o = b \wedge \Diamond_e o \neq b$$

(Here o translates 'the CEO of Tesla.') For if this sentence is true in our context c, then at least one of the two descriptions that figure in that sentence must be non-rigid over the class of worlds over which 'might' quantifies in c.[5] Of course, we could in principle square this point with the claim that 'the inventor of bitcoin' is rigid over $\mathcal{B}_c$ by maintaining that 'the CEO of Tesla' is non-rigid over that class of worlds. But this fails to avoid the problem, since it also appears that (7) is unambiguously false in our context:

(7) The CEO of Tesla might not be the CEO of Tesla.

Kripke-style considerations will again lead us from this observation to the conclusion that 'the CEO of Tesla' is rigid over $\mathcal{B}_c$, and so our problem returns.

So the unambiguous falsity of (3) and (7) in our context suggest that the descriptions figuring in those sentences are both epistemically *rigid* in our context. But the apparent truth of (6) in our context suggests instead that at least one of those descriptions is epistemically *non-rigid* in our context.

This puzzle concerns definite descriptions, but essentially the same puzzle arises in connection with proper names. To see this, consider the following sentence:

(8) Elon Musk might be Satoshi Nakamoto, but (then again) Elon Musk might not be Satoshi Nakamoto.

$$\Diamond_e m = n \wedge \Diamond_e m \neq n$$

(Here m translates 'Elon Musk' and n 'Satoshi Nakamoto.') More news about bitcoin: 'Satoshi Nakamoto' is the name used by the unknown person who created bitcoin. That is, the person who invented bitcoin has published whitepapers and blog posts under that name, but it is not a matter of public knowledge which legal person is picked out by that name. So it seems to me that (8) is true in our present context, since (a) we don't know who Satoshi Nakamoto is, nor is it a matter of public knowledge who he or she is, and (b) Elon Musk is not a wholly implausible candidate for being Satoshi Nakamoto.

For reasons parallel to the ones given above in our discussion of (6), the truth of (8) in our present context c suggests that at least one of the two names

[5] To see this, suppose, for *reductio*, that (6) is true relative to c and that both o and b are epistemically rigid over $\mathcal{B}_c$. Then since $\Diamond_e o = b$ is true at c, there is a world $w \in \mathcal{B}_c$ at which o and b pick out the same object r. But then since o and b are rigid over $\mathcal{B}_c$, it follows that, for any world $w' \in \mathcal{B}_c$, the referent of o at w' will be r and the referent of b at w' will be r. Thus, $o \neq b$ will be false at w'. Since this holds for an arbitrary world in $\mathcal{B}_c$, it holds for them all, which means $\Diamond_e o \neq b$ is false at c. But this contradicts the supposition that (6) is true at c.

that figure in that sentence is non-rigid over $\mathcal{B}_c$. But that result seems to be in tension with the fact that the following sentences appear to be unambiguously false (in our context):

(9) Elon Musk might not be Elon Musk.

(10) Satoshi Nakamoto might not be Satoshi Nakamoto.

For the falsity of the *de re* forms of (9) and (10) in our context suggests that these two names are both rigid over $\mathcal{B}_c$.

Now one might try to get around the proper names version of this puzzle by denying that the correct response to the truth of (8) in c is to conclude that one of the proper names in question is non-rigid over $\mathcal{B}_c$. "Millians" about proper names would presumably maintain that both of those names are rigid over $\mathcal{B}_c$.[6] This would allow them to give a straightforward, Kripkean explanation of the unambiguous falsity of (9) and (10). But such a move would then require giving an alternative explanation of the apparent truth of (8) in the present context. I won't pursue this strategy further here, for even if it were carried off successfully, it would leave untouched our puzzle about definite descriptions, and so the general problem would still be with us.

In what follows, I use the foregoing puzzles to examine theories of modals, singular terms, and variables. I begin in §2 by examining a theory according to which variables range over *individual concepts* rather than over individuals. Although this theory does resolve the puzzle about definite descriptions, it does so only by upending Kripke's account of the interaction between definite descriptions and metaphysical modals. That highlights an important constraint on an adequate solution to our puzzles: it should be compatible with Kripke's initial observations about the interaction between singular terms and metaphysical modals.

In §3, I develop a version of *dynamic semantics* that resolves our puzzles while respecting Kripke's observations. Building on earlier work (Groenendijk et al., 1996), I extend the dynamic approach to languages containing an abstraction operator and a metaphysical possibility modal. An issue concerning the treatment of proper names leads us to consider two versions of dynamic semantics. One requires us to appeal to a class of epistemically possible worlds that are not metaphysically possible, thus effectively bifurcating modal space (cf. Soames, 2009) (§3.3). The other avoids this bifurcation by incorporating *two-dimensionalist* ideas into the dynamic setting (§3.4). I close in §4 with a brief discussion of the prospects of alternative static approaches, including static versions of two-dimensionalism.[7]

[6] For defenses of Millianism, see Salmon (1986) and Soames (2002), among others.

[7] Yalcin (2015) also uses the infelicity of sentences like (3), along with other data, to motivate a version of dynamic semantics similar to the one presented here. But Yalcin's focus is on the interaction between epistemic modals and quantifiers, and he treats definite descriptions along the quantificational model of Russell (1905). Thus, questions about the rigidity and scope of singular terms—questions that constitute the topic of the present essay—do not arise for Yalcin. Also, Yalcin does not discuss metaphysical modals in detail, whereas metaphysical modality has an important role to play in our discussion. Formally, these differences reveal

2 Individual concepts

Let's assume that we're not going to abandon the epistemic non-rigidity of descriptions like 'the inventor of bitcoin.' This allows us to maintain that (6) is true in our context.

(6) The CEO of Tesla might be the inventor of bitcoin, but (then again) the CEO of Tesla might not be the inventor of bitcoin.

$\Diamond_e o = b \wedge \Diamond_e o \neq b$

Then the basic trouble is to understand how the *de re* form of (3) might be false in our context despite the fact that 'the inventor of bitcoin' is non-rigid over $\mathcal{B}_c$.

(3) The inventor of bitcoin might not be the inventor of bitcoin.

De re form: $(\lambda x.\Diamond_e x \neq b)(b)$

Now, the standard way of evaluating this logical form with respect to our context c would be to find the referent r of b at the actual world v, and then check to see whether there is a world $v' \in \mathcal{B}_c$ at which r is distinct from the referent r' of b at v. Note that according to this procedure for evaluating $(\lambda x.\Diamond_e x \neq b)(b)$, the λ-abstract $(\lambda x.\Diamond_e x \neq b)$ is, so to speak, "looking for" an individual. The individual we hand it in the course of our evaluation is the individual to whom b refers in the actual world. If this is Elon Musk, then we are handing that λ-abstract Elon Musk. The resulting logical form is then predicted to be true in our context since, for all we know, Elon Musk is not the inventor of bitcoin.

Now, a tempting thought is that, rather than handing that λ-abstract the *referent* or *extension* of b at the actual world, we should hand it the *sense* or *intension* of b. Formally, we can think of the intension of b as an *individual concept*, a function from worlds to individuals. Since b translates 'the inventor of bitcoin,' the intension of b is the function i_b that maps each world w to the inventor of bitcoin at w.[8] We would then assess the sentence by finding, not the extension r of b at the actual world, but the *intension* i_b of b, and then checking to see whether there is a world $v' \in \mathcal{B}_c$ such that i_b maps v' to an individual that is distinct from the referent of b at v'. And we know that there will be no such world. This is because, for each world v', the intension i_b of b maps v' to the referent r' of b at v'—this is just what it means to be the intension of b. Thus, such a theory will predict that the *de re* logical form of (3) is false at every context; it will thus predict the unambiguous falsity of (3).

themselves in the fact that, unlike Yalcin's semantics, our semantics is defined for a language that includes individual constants, an abstraction operator, and a metaphysical possibility modal.

The discussion of Aloni (2001, Ch. 3) is also relevant, though she uses sentences like (3) not to motivate dynamic semantics *per se*, but to constrain her theory of quantified epistemic modality. And like Yalcin's, Aloni's semantics is defined for a language that contains neither an abstraction operator nor a metaphysical possibility modal. I discuss Aloni's approach in more detail in Ninan (2018).

[8] We continue to ignore the possibility that some worlds may fail to have a unique inventor of bitcoin.

Note that this argument makes no assumption as to whether or not b is rigid across $\mathcal{B}_c$. This is because the referent r of b at the actual world never enters into our evaluation of the formula. Thus, whether there is a world v' in $\mathcal{B}_c$ at which the referent r' of b is distinct from r is simply irrelevant to the truth value of the formula.

One way to work this idea out out in detail is to have variables range over individual concepts rather than over individuals. Such systems are often called 'systems of contingent identity' and trace back to Carnap (1947).[9] On this sort of approach, a variable assignment maps each variable to an individual concept, rather than to an individual. So if g is a variable assignment, x a variable, and v a world, $g(x)$ is an individual concept, and $g(x)(v)$ an individual.

Given a variable assignment g, let us say that g' is an *x-variant of g* just in case for all variables y distinct from x, $g'(y) = g(y)$. Then given a variable assignment g and a singular term a, let $g[x/i_a]$ be the x-variant of g that maps x to i_a (the intension of a). Then the clause for the abstraction operator in such a system would presumably look like this:

$$[\![(\lambda x.\phi)(a)]\!]^{c,v,g} = 1 \text{ iff } [\![\phi]\!]^{c,v,g[x/i_a]} = 1$$

And on this sort of approach, the formula $x \neq a$ would presumably be associated with the following truth-conditions:

$$[\![x \neq a]\!]^{c,v,g} = 1 \text{ iff } g(x)(v) \neq i_a(v)$$

This yields the result that sentences like (3) and (7) are true at no point of evaluation. For consider the following:

$$[\![(\lambda x.\Diamond_e x \neq a)(a)]\!]^{c,v,g} = 1 \text{ iff}$$
$$[\![\Diamond_e x \neq a]\!]^{c,v,g[x/i_a]} = 1 \text{ iff}$$
$$\text{there is a world } v' \in \mathcal{B}_c \text{ such that } [\![x \neq a]\!]^{c,v',g[x/i_a]} = 1 \text{ iff}$$
$$\text{there is a world } v' \in \mathcal{B}_c \text{ such that } g[x/i_a](x)(v') \neq i_a(v').$$[10]

The crucial point to note here is that $g[x/i_a](x)$ is simply i_a. Thus, the sentence will be true at this point of evaluation just in case:

$$\text{there is a world } v' \in \mathcal{B}_c \text{ such that } i_a(v') \neq i_a(v').$$

But of course there can be no world v' such that $i_a(v')$ is distinct from $i_a(v')$. So sentences like (3) and (7) are predicted to be inconsistent (i.e. false at every point of evaluation) on this approach. Sentence (6), on the other hand, will be true at some points of evaluation since the proposal allows that descriptions like 'the inventor of bitcoin' may be non-rigid over the set of worlds over which epistemic 'might' quantifies.

[9] See also Bressan 1972, Gibbard 1975, Aloni (2001, 2005), Fitting 2004, Garson (2006), and Holliday and Perry 2014.

[10] The recursive clause for $\Diamond_e$ should likely make its interpretation sensitive to the evaluation world v, but I gloss over this subtlety in what follows. I never consider cases in which a modal is embedded under another intensional operator.

These are nice results. This approach seems to achieve scopelessness without rigidity: definite descriptions like 'the inventor of bitcoin' may be non-rigid across $\mathcal{B}_c$ despite being scopeless with respect to epistemic 'might.' Unfortunately, there is a problem with this approach.

The problem is that the mechanism by which this approach achieves the scopelessness of descriptions is too general. As a result, it ends up predicting that definite descriptions are scopeless not just with respect to epistemic modals, but with respect to *all* modals. And this means that the theory upends Kripke's pleasing story about the interaction between singular terms and metaphysical modals.

The main issue is that this approach predicts that the *de r* form of sentences like (2) are also false:

(2) The President of the U.S. in 2019 might not have been the President of the U.S. in 2019.

De re form: $(\lambda x.\Diamond_m x \neq p)(p)$

For if the variable x ranges over individuals concepts, then $(\lambda x.\Diamond_m x \neq p)$ is looking for an individual concept. If we then hand it the intension i_p of p, the resulting formula will be true at the actual world v just in case there is a (metaphysically accessible) world v' at which $i_p(v')$ is distinct from the referent of p at v'. But, again, there is no such world, since, for each world v', i_p maps v' to the referent of p at v'—again, that's just how the intension of p is defined. Thus, (2) will be false on its *de re* reading. Since we agreed earlier that it was false on its *de dicto* reading, it will be false on both readings. Thus, the present theory will no longer predicts the contrast between (1) and (2) with which we began, for it will predict that both (1) and (2) are unambiguously false.

The difference between sentences (2) and (3) reveals an asymmetry between how definite descriptions interact with epistemic modals versus how they interact with metaphysical modals. The individual concepts theory fails to respect this asymmetry. We need an approach that posits a deeper semantic difference between epistemic and metaphysical modals.

3 Dynamic semantics

In this section, I make the case that a certain version of dynamic semantics can solve our two puzzles, while at the same time respecting Kripke's observation about sentences (1) and (2). The version of dynamic semantics considered here is based on the theory of Groenendijk et al. (1996), though their system is extended to a language that contains an abstraction operator and a metaphysical possibility modal. Also, since we are not concerned with the topic of intersentential anaphora, we can adopt a somewhat simpler version of their approach. In what follows, I shall discuss the implications of this theory for our two puzzles, and for Kripke's contrast; a more general presentation of the semantics can be found in the appendix. I'll begin by focusing on our two puzzles (§3.1), before turning to the treatment Kripke's contrast (§§3.2–3.4).

3.1 The two puzzles

In dynamic semantics, sentences are not assigned truth values relative to a sequence of parameters (at least not if the parameters contain only the usual suspects, such as context, world, variable assignment, etc.). Rather, the meaning of a sentence is understood in terms of its capacity to update a state of information. Meanings are not truth-conditions, but "context-change potentials." A *context* or *state of information* is typically represented by a set of possibilities, where a *possibility* is a pair of a possible world and a variable assignment. The meaning of a sentence is then represented as a function from states of information to states of information. We write $s[\phi]$ for the result of applying the meaning of ϕ to the argument s; $s[\phi]$ can be thought of as the state of information that results from updating s with ϕ.

A state of information s is said to *support* a sentence ϕ just in case updating s with ϕ returns s, in symbols, $s[\phi] = s$. Where ϕ contains no epistemic modal, a state of information will support ϕ just in case ϕ is true at every possibility in the state. Although sentences cannot in general be evaluated for truth with respect to a possibility, sentences drawn from the epistemic-modal-free fragment of the language can be so evaluated (in the particular dynamic theory under discussion), and I will be appealing to this notion below.

Now, I would like to say that if an agent's state of information supports a sentence ϕ, then she is (epistemically speaking) in a position to assert ϕ. But in possible worlds semantics, it is standard to represent an agent's state of information by a set of *possible worlds*, not by a set of *possibilities* (world-assignment pairs). And, as we have defined it, *support* is a relation between sentences and sets of possibilities. But we can get around this problem by defining a corresponding support relation between sets of *possible worlds* and sentences as follows:

> A set σ of possible worlds worlds *supports* a sentence ϕ just in case: there is a variable assignment g such that $\{\langle v, g\rangle : v \in \sigma\}$ supports ϕ.

This allows us to say that if an agent's state of information—the set of possible worlds compatible with what she knows—supports a sentence ϕ, then she is (epistemically speaking) in a position to assert ϕ.[11] (I use the terms 'support' and 'state of information' ambiguously in what follows; context should disambiguate).

If the result of updating *every* state of information with ϕ is the empty set—if $s[\phi] = \emptyset$, for all s—then we say that ϕ is *inconsistent*.

Let's start by considering our puzzle about the interaction between epistemic modals and definite descriptions. Our aim is to obtain two results. First, we want to show that sentence (6) is assertable for us, given our present state of information. Second, we want to show that sentences of the form $(\lambda x.\Diamond_e x \neq$

[11] This is way of implementing the idea that knowledge is the norm of assertion within the dynamic framework. See Gazdar (1979), Unger (1979), and Williamson (1996, 2000) for discussion of the knowledge norm.

$a)(a)$ are inconsistent, where a is any singular term. This second result will ensure that both (3) and (7) are inconsistent on their *de re* readings.

(6) The CEO of Tesla might be the inventor of bitcoin, but (then again) the CEO of Tesla might not be the inventor of bitcoin.

$\Diamond_e o = b \wedge \Diamond_e o \neq b$

(3) The inventor of bitcoin might not be the inventor of bitcoin.

De re form: $(\lambda x.\Diamond_e x \neq b)(b)$

(7) The CEO of Tesla might not be the CEO of Tesla.

De re form: $(\lambda x.\Diamond_e x \neq o)(o)$

Let's start with the first task, predicting that we are in a position to assert (6). In dynamic semantics, an epistemically modalized formula $\Diamond_e \phi$ "tests" a state of information s for compatibility with ϕ (Veltman, 1996). If s is compatible with ϕ, s passes the test, and the updating procedure returns s unchanged; if s is not compatible with ϕ, s fails the test, and the updating procedure "crashes" the context and returns the empty set:

$$s[\Diamond_e \phi] = \begin{cases} s & \text{if } s[\phi] \neq \emptyset; \\ \emptyset & \text{otherwise.} \end{cases}$$

Where ϕ itself contains no occurrences of $\Diamond_e$, this procedure simply amounts to checking whether there is a possibility in s at which ϕ is true. Consider, in particular, the formula $\Diamond_e o = b$, the first conjunct of (6). Updating a state of information s with this formula will return s if there is a possibility i in s at which the referent of o at i is identical to the referent of b at i; otherwise, it returns the empty set. Similarly, updating a state s with $\Diamond_e o \neq b$ will return s if there is a possibility i in s at which the the referent of o at i is distinct from the referent of b at i; otherwise, it returns the empty set.

Fact 1. *For any state s:*

(i) $$s[\Diamond_e o = b] = \begin{cases} s & \textit{if there is an } i \in s \textit{ such that } i(o) = i(b); \\ \emptyset & \textit{otherwise.} \end{cases}$$

(ii) $$s[\Diamond_e o \neq b] = \begin{cases} s & \textit{if there is an } i \in s \textit{ such that } i(o) \neq i(b); \\ \emptyset & \textit{otherwise.} \end{cases}$$

(For any singular term a and any possibility $i = \langle v, g \rangle$, $i(a)$ is the referent of a at v.)

The update of a state s with a conjunction $\phi \wedge \psi$ proceeds by first updating s with ϕ, and then updating the resulting state, $s[\phi]$, with ψ:

$$s[\phi \wedge \psi] = s[\phi][\psi]$$

Thus, the result of updating a state s with (6) will be:

$$s[\Diamond_e o = b][\Diamond_e o \neq b]$$

Together with **Fact 1**, this gives us the following result:

Fact 2. *For any state s:*

(i) $s[(6)] = s$ *if there is an $i \in s$ such that $i(o) = i(b)$, and there is an $i \in s$ such that $i(o) \neq i(b)$*

(ii) $s[(6)] = \emptyset$ *otherwise.*

Now, think about our state of information. We don't know whether or not the CEO of Tesla (o) is the inventor of bitcoin (b). So there are possibilities i compatible with what we know in which the CEO of Tesla is the inventor of bitcoin, and possibilities i' compatible with what we know in which the CEO of Tesla is not the inventor of bitcoin. Possibilities i of the first kind will be such that $i(o) = i(b)$, for $i(o)$ is the CEO of Tesla at i, and $i(b)$ is the inventor of bitcoin at i. Possibilities i' of the second kind will be such that $i'(o) \neq i'(b)$. Thus, from **Fact 2**, it follows that our state of information s supports (6), which means that we are in a position to assert it.

Let's turn now to the second part of our puzzle, the inconsistency of sentences of the form $(\lambda x.\Diamond_e x \neq a)(a)$, where a is any singular term. Recall that $\phi(a/x)$ is the result of substituting the singular term a for every free occurrence of x in ϕ. Then given a *de re* form $(\lambda x.\Diamond_e \phi)(a)$, we may say that $\Diamond_e \phi(a/x)$ is its *de dicto counterpart*. According to the version of dynamic semantics adopted here, a *de re* form that does not contain a metaphysical modal is equivalent to its *de dicto* counterpart, in the sense that both are associated with the same update:

Fact 3. *If ϕ does not contain a $\Diamond_m$, then for any state s and any singular term a:*

$$s[(\lambda x.\Diamond_e \phi)(a)] = s[\Diamond_e \phi(a/x)]$$

(A proof of this equivalence may be found in the appendix.) This is presumably what it means to say, in the dynamic setting, that singular terms are scopeless with respect to epistemic modals.

Now since $x \neq a$ does not contain a metaphysical modal, this means that the *de re* form:

$$(\lambda x.\Diamond_e x \neq a)(a)$$

is equivalent to its *de dicto* counterpart:

$$\Diamond_e a \neq a$$

But since the latter is obviously inconsistent—there is no possibility i in any state s at which $i(a) \neq i(a)$—so is the former. So sentences of the form $(\lambda x.\Diamond_e x \neq a)(a)$, where a is any singular term, are inconsistent. Since the *de re* forms of (3) and (7) are of the requisite form, these are predicted to be inconsistent, as desired.

That takes care of our puzzle concerning epistemic modals and definite descriptions: (6) is predicted to be assertable by us, while (3) and (7) are predicted to be inconsistent. But what about the corresponding puzzle concerning epistemic modals and proper names? The puzzle about names, recall, requires us to reconcile the assertability of (8) with the inconsistency of the *de re* forms of (9) and (10):

(8) Elon Musk might be Satoshi Nakamoto, but (then again) Elon Musk might not be Satoshi Nakamoto.

$\Diamond_e m = n \wedge \Diamond_e m \neq n$

(9) Elon Musk might not be Elon Musk.

De re form: $(\lambda x.\Diamond_e x \neq m)(m)$

(10) Satoshi Nakamoto might not be Satoshi Nakamoto.

De re form: $(\lambda x.\Diamond_e x \neq n)(n)$

Fact 3 holds for any singular term a, and so holds even when a is a proper name. Thus, it guarantees the inconsistency of the *de re* forms of (9) and (10).

But to predict the assertability of (8) for us, we must accept that one of the names 'Elon Musk' (m) and 'Satoshi Nakomoto' (n) is non-rigid over our state of information s. For if they are both rigid over s, then either every possibility i in s will be such that $i(m) = i(n)$, or every possibility i in s will be such that $i(m) \neq i(n)$. Either way, s will end up failing to support one of the conjuncts of (8). To see this, it suffices to note that the **Fact 2** still holds even when o is replaced by m and b is replaced by n. So let's assume for the moment that at least one of the names m, n is non-rigid over s; we'll re-visit this assumption later (see §3.3).

So we may assume that there are possibilities i in s such that $i(m) = i(n)$, and possibilities i' in s such that $i(m) \neq i(n)$. Within this framework, this is simply to assume that we don't know whether or not Elon Musk is Satoshi Nakomoto. Given the relevant analogue of **Fact 2**, this will mean that we are position to assert sentence (8).

3.2 Descriptions and metaphysical modality

So dynamic semantics solves our initial two puzzles. But so far we've only reproduced the results of the individual concepts theory of §2 using somewhat more elaborate machinery. We need to reassure ourselves that the theory preserves Kripke's original observations about sentences (1) and (2), repeated below.

(1) Trump might not have been Trump.

(2) The President of the U.S. in 2019 might not have been the President of the U.S. in 2019.

This turns out to be more straightforward in the case of sentence (2)—the case of descriptions—so we deal with that case first before turning to sentence (1) and the case of proper names.

We're assuming that sentence (2) has two forms, a *de re* form and a *de dicto* form:

De re form of (2): $(\lambda x.\Diamond_m x \neq p)(p)$

De dicto form of (2): $\Diamond_m p \neq p$

Now the *de dicto* form is obviously inconsistent, so we do not want the analogue of **Fact 3** to hold for metaphysical modals, for we want to predict that the *de re* form of (2) is assertable. We want the *de re–de dicto* distinction to collapse in the epistemic case, but not in the metaphysical case, at least not when we are dealing with definite descriptions. This was the point at which the individual concepts theory faltered.

As I mentioned above, a sentence that does not contain an epistemic modal can be said to be true or false with respect to a possibility. This holds for sentences like $(\lambda x.\Diamond_m x \neq p)(p)$ since they do not contain epistemic modals. According to the dynamic theory adopted here, we evaluate a sentence like this at a possibility i in essentially the same manner we proposed in §1. So to evaluate $(\lambda x.\Diamond_m x \neq p)(p)$ at i, we first find the referent r of p at i, and then ask if there is a possibility i' at which r is distinct from the referent r' of p at i'. Now p is the description 'the President of the U.S. in 2019,' so it is presumably non-rigid. As our discussion in §1 revealed, this is sufficient to guarantee that $(\lambda x.\Diamond_m x \neq p)(p)$ is true at any possibility i. And if $(\lambda x.\Diamond_m x \neq p)(p)$ is true at every possibility i, then it is supported by every state s, i.e. $s[\Diamond_m x \neq p)](p) = s$. And this ensures that we are in a position to assert the *de re* form of (2).

Note that this argument assumes that the metaphysical modal $\Diamond_m$ quantifies over the space of all possibilities.[12] This assumption comes in when we asked whether *there is a possibility* i' at which the referent r of p at i is distinct from the referent r' of p at i'. Note the unrestricted nature of that quantifier. If $\Diamond_m$ did not quantify over the space of all possibilities, we would instead have to ask whether *there is a possibility* i' *metaphysically accessible from* i that has the relevant property. Since we will have occasion to re-visit the assumption that $\Diamond_m$ quantifies over the space of all possibilities below, it will be useful to state the result concerning the *de re* form of (2) in a slightly more general way.

Let $\mathcal{R}$ represent the metaphysical accessibility relation, a binary equivalence relation on the space of possible worlds. And let $\mathcal{R}(i)$ be defined as follows:

If $i = \langle v, g\rangle$, then $\mathcal{R}(i) = \{\langle v', g'\rangle : g' = g \text{ and } v\mathcal{R}v'\}$.

[12] This is slightly misleading, but not in a way that affects any issue of substance. What the argument really assumes that the metaphysical accessibility relation, $\mathcal{R}$, is the universal relation over possible worlds. Technically, at each possibility i, $\Diamond_m$ quantifies over $\mathcal{R}(i)$, but $\mathcal{R}(i)$ is not the space of all possibilities. For if $i = \langle v, g\rangle$ and $g' \neq g$, then no possibility of the form $\langle v', g'\rangle$ will be in $\mathcal{R}(i)$. So strictly speaking, the assumption in question is that $\mathcal{R}(i)$ includes all possibilities $\langle v', g'\rangle$ where $g' = g$. But the simplification in the text is harmless, and I stick with it for ease of exposition.

And let us say that a singular term a is *rigid over a set s of possibilities* just in case there are possibilities $i, i' \in s$ such that the referent of a at i is distinct from the referent of a at i; if this condition does not hold, a is *non-rigid over s*. Then we have:

Fact 4. *Let s be a state such that, for each possibility i in s, a is non-rigid over $\mathcal{R}(i)$. Then:*

$$s[(\lambda x.\Diamond_m x \neq a)(a)] = s.$$

A proof of this fact can be found in the appendix.

3.3 Names and metaphysical modality

So the dynamic theory improves on the individual concepts theory. Both approaches solve our initial two puzzles (at least given the assumption that names are epistemically non-rigid), but the individual concepts approach runs into trouble with sentence (2), incorrectly predicting that it has no acceptable reading. The dynamic approach, on the other hand, predicts that that sentence is assertable in our context on its *de re* reading.

But there is one outstanding issue: the status of sentence (1), 'Trump might not have been Trump.' We began the essay with Kripke's contrast between (1), which has no acceptable reading, and (2), which does. Dynamic semantics predicts the latter; can it also predict the former?

The particular dynamic theory we've been discussing faces a *prima facie* difficulty here. The difficulty arises from two assumptions. The first assumption is one we were just discussing above, the assumption that the metaphysical modal quantifies over the space of all possibilities, i.e. the assumption that $\mathcal{R}$ is the universal relation on the space of all possible worlds. The second assumption is that not all proper names are rigid over our state of information s.

Here's the problem. Assume that we want all sentences like (1) to come out inconsistent. This is simply the scopelessness of proper names with respect to metaphysical modals that we discussed earlier. Note that from our second assumption it follows that some proper name a is non-rigid over our state of information s. So there are possibilities i and i' in s such that $i(a) \neq i'(a)$.

Now since i and i' are possibilities in s, they are in the space of all possibilities. It follows that a is non-rigid over the space of all possibilities. Now if $\mathcal{R}$ is the universal relation over the space of possibilities, then, where i is any possibility, $\mathcal{R}(i)$ is simply the space of all possibilities. So it follows that a is non-rigid over $\mathcal{R}(i)$, for any possibility i.

But recall **Fact 4** from above. Given that a is non-rigid over $\mathcal{R}(i)$, for any possibility i, it follows that for any possibility i in any state s, a is non-rigid over $\mathcal{R}(i)$. Given **Fact 4**, this means that every state s supports $(\lambda x.\Diamond_m x \neq a)(a)$, which means that we should be in a position to assert it. But this is the wrong result, for a is a proper name. Thus, $(\lambda x.\Diamond_m x \neq a)(a)$ is the *de re* form of a sentence like (1), 'Trump might not be have been Trump.' But we are not in a position to assert such sentences; they ought to come out inconsistent, no matter how they are parsed.

Why should think that some proper names are non-rigid over our state of information s? The motivation for this goes back to the discussion of our puzzle concerning proper names and epistemic modals in §3.1. Recall that we noted there that sentence (8) will be supported by our information state s only if one of the names that figures in that sentence is non-rigid over s.

(8) Elon Musk might be Satoshi Nakamoto, but (then again) Elon Musk might not be Satoshi Nakamoto.

$\Diamond_e m = n \wedge \Diamond_e m \neq n$

Thus, if we want the result that we are in a position to assert (8), then we're going to have to accept that either 'Elon Musk' or 'Satoshi Nakomoto' is non-rigid over s. And if a singular term is non-rigid over s, it will be non-rigid over the space of all possibilities. And if we accept that, then, given the universality of $\mathcal{R}$, we'll have to accept that we are either in a position to assert the *de re* form of (11) or in a position to assert the *de re* form of (12):

(11) Elon Musk might not have been Elon Musk.

$(\lambda x.\Diamond_e x \neq m)(m)$

(12) Satoshi Nakamoto might not have been Satoshi Nakamoto.

$(\lambda x.\Diamond_e x \neq n)(n)$

But then we have to repudiate Kripke's claim that names are scopeless with respect to metaphysical modals.

This problem arises from the two assumptions mentioned above, the universality of $\mathcal{R}$, and the non-rigidity of some proper names. Accordingly, one can resist the problem by denying either one of these assumptions.

If one denies the second assumption and instead maintains that all proper names are rigid over the space of all possibilities, then the *de re* forms of sentences like (1), (11), and (12) will be predicted to be inconsistent by our dynamic theory. This follows from another fact about our dynamic theory:

Fact 5. *Let s be a state such that, for each possibility i in s, a is rigid across $\mathcal{R}(i)$. Then:*

$$s[(\lambda x.\Diamond_m x \neq a)(a)] = \emptyset.$$

(See the appendix for a proof of this.) This is the sort of approach a "Millian" about names might endorse. If we took this approach, we would then need to explain the apparent acceptability of sentences like (8) in some other way. So while this avoids the present problem, it remains to be seen how the the apparent acceptability of (8) will be explained. I set this approach aside here.

A second response would be to accept the assumption that names are not rigid across the whole space of possibilities, but then deny the assumption that $\mathcal{R}$ is universal. From the point of view of certain possibilities, certain other possibilities are not metaphysically accessible. How would this help? Well, if names need not be rigid across the whole space of possibilities, then we could have a state s that contains possibilities i and i' such that $i(m) = i(n)$ and $i'(m) \neq i'(n)$. Such a state would support (8), and thus predict the potential assertability of that sentence.

But how would this approach predict the inconsistency of sentences like (1)? Well, suppose we adopted the following constraint:

For each possibility i, and each name a, a is rigid over $\mathcal{R}(i)$.

If this constraint is adopted, then sentences like (1) will turn out to be inconsistent. This follows again from **Fact 5**, mentioned above.

Note a feature of this second approach. Suppose that s represents our state of information, that i is a possibility in s such that $i(m) = i(n)$, and that i' is a possibility in s such that $i'(m) \neq i'(n)$. On the present proposal, every possibility i'' metaphysically accessible from i will be such that $i''(m) = i''(n)$. So that means that i' is not metaphysically accessible from i. How should we interpret this situation? The precise description of the situation is somewhat delicate, given the dynamic setting in which we are operating. But it is clear that this picture has affinities with the idea that there are epistemic possibilities that are not metaphysically possible (Soames, 2009). For it seems that, from the point of view of i, i' is epistemically, but not metaphysically, possible.

Now, the idea that there are epistemic possibilities that are not metaphysically possible might sound like a mere truism to some, especially if one considers mathematical or logical falsehoods which we do not know to be false. I am sympathetic to this view. But when it comes to the sorts of issues we are dealing with—which involve empirical ignorance of identity claims—there is a tradition of thinking such cases can be handled without bifurcating modal space in this way. I have in mind the *two-dimensionalist* approach associated with Chalmers and Jackson (among others).[13] Perhaps a two-dimensional version of dynamic semantics could resolve our problem with sentences like (1), and could do so without requiring us to appeal to a class of epistemically possible worlds that are not metaphysically possible. I argue below that this is indeed the case.

3.4 Two-dimensional dynamic semantics

Our challenge is to reconcile the fact that identity statements involving names may be epistemically contingent, even though names are scopeless with respect to metaphysical modals. For example, we would like to secure the assertability of (8) in our context, while also maintaining the inconsistency of (11) and (12).

(8) Elon Musk might be Satoshi Nakamoto, but (then again) Elon Musk might not be Satoshi Nakamoto.

$\Diamond_e m = n \wedge \Diamond_e m \neq n$

[13] See Davies and Humberstone (1980), Jackson (1998), Chalmers (2006a,b), and Schroeter (2017) among others.

(11) Elon Musk might not have been Elon Musk.

$(\lambda x.\Diamond_e x \neq m)(m)$

(12) Satoshi Nakamoto might not have been Satoshi Nakamoto.

$(\lambda x.\Diamond_e x \neq n)(n)$

Standard versions of static two-dimensionalism yield this result, if they treat variables as ranging over individuals. One of the key features that enables them to do this is they associate with each proper name a *two-dimensional intension*, or a function from pairs of worlds to individuals. For example, we might have:

> $\mathcal{I}(m)$ is the function that maps a pair of worlds (v, v') to the individual called 'Elon Musk' in v.
>
> $\mathcal{I}(n)$ is the function that maps a pair of worlds (v, v') to the individual called 'Satoshi Nakomoto' in v.

Note two features of $\mathcal{I}(m)$ ($\mathcal{I}(n)$ has both features as well). First, $\mathcal{I}(m)$ is sensitive to the first world argument it takes. That is, there are v, v', v'' such that $\mathcal{I}(m)(v, v'') \neq \mathcal{I}(m)(v', v'')$. In two-dimensional semantics, formulas are evaluated with respect to a pair of worlds, and this feature of $\mathcal{I}(m)$ enables the semantic value of a name to be shifted by operators that shift the first world of evaluation. Epistemic modals shift both worlds of evaluation, which means that names behave non-rigidly when they interact with epistemic modals:

$$[\![\Diamond_e \phi]\!]^{c,v,v',g} = 1 \text{ iff there is a world } v'' \in \mathcal{B}_c \text{ such that } [\![\phi]\!]^{c,v'',v'',g} = 1$$

Second, note that, for any worlds v, v', v'', $\mathcal{I}(n)(v, v') = \mathcal{I}(n)(v, v'')$. I will record this feature by saying that $\mathcal{I}(n)(v)$ *is a constant function*, for any world v. This allows names to behave rigidly when they interact with metaphysical modals, since metaphysical modals only shift the second world of evaluation:

$$[\![\Diamond_m \phi]\!]^{c,v,v',g} = 1 \text{ iff there is a world } v'' \text{ such that } [\![\phi]\!]^{c,v,v'',g} = 1$$

These ideas can be implemented in dynamic semantics, if we make two changes to the theory. First, we associate with singular terms with two-dimensional intensions; proper names are associated two-dimensional intensions like the ones above (we discuss descriptions below). Second, we take possibilities to be triples $\langle v, v', g\rangle$ consisting of a world v, a world v', and a variable assignment g. States of information, in the technical sense, will be sets of such possibilities. A state of information s will support a sentence ϕ just in case $s[\phi] = s$. And we can again define a more intuitive notion of support:

> A set σ of possible worlds supports a sentence ϕ just in case there is a variable assignment g such that $\{\langle v, v, g\rangle : v \in \sigma\}$ supports ϕ.

Otherwise, the semantics remains largely the same, save for the treatment of the metaphysical modal. Since the semantics is otherwise the same, the treatment of our initial two puzzles is preserved when we move to the two-dimensional version of dynamic semantics (see appendix for details). In particular, this approach preserves the assertability of (8) in our context.

The difference comes when we turn to metaphysical modality. Let's examine how this approach predicts Kripke's contrast between (1) and (2). Let's start by examining sentence (11), which is of the same form as (1).

(11) Elon Musk might not have been Elon Musk.

$(\lambda x.\Diamond_e x \neq m)(m)$

On the two-dimensional dynamic proposal, possibilities are triples of the form $\langle v, v', g\rangle$. According to this approach, $(\lambda x.\Diamond_m x \neq m)(m)$ will be true at a possibility $\langle v, v', g\rangle$ just in case there is a possibility $\langle v, v'', g\rangle$ such that the extension of m at (v, v') is distinct from the extension of m at (v, v''). (Note that the metaphysical modal, in essence, only shifts the second world of the initial possibility $\langle v, v', g\rangle$.) But the extension of m at (v, v') is the individual called 'Elon Musk' at w. And the extension of m at (v, v''), for any world v'', is also the individual called 'Elon Musk' at v. So there is no possibility $\langle v, v'', g\rangle$ such that the extension of m at (v, v') is distinct from the extension of m at (v, v''). It follows that $(\lambda x.\Diamond_m x \neq m)(m)$ is false at the possibility $\langle v, v', g\rangle$. Since $\langle v, v', g\rangle$ was an arbitrary possibility, it follows that $(\lambda x.\Diamond_m x \neq m)(m)$ is false at every possibility. And from this it follows that $(\lambda x.\Diamond_m x \neq m)(m)$ is inconsistent.

Here is the general result:

Fact 6. *Let a be a singular term such that, for any world v, $\mathcal{I}(a)(v)$ is a constant function from worlds to individuals. Then for any state s:*

$$s[(\lambda x.\Diamond_m x \neq a)(a)] = \emptyset$$

Thus, our two-dimensional version of dynamic semantics predicts Kripke's observation about sentences like (1), (11), and (12).

The account still respects Kripke's observation about sentence (2):

(2) The President of the U.S. in 2019 might not have been the President of the U.S. in 2019.

For we also have the following result:

Fact 7. *Let a be a singular term such that, for any world v, $\mathcal{I}(a)(v)$ is not a constant function. Then for any state s:*

$$s[(\lambda x.\Diamond_m x \neq a)(a)] = s$$

Assume the description 'the President of the U.S. in 2019' (p) is associated with the following two-dimensional intension:

> $\mathcal{I}(p)$ is the function that maps a pair of worlds (v, v') to the President of the U.S. in 2019 in v'.

Then **Fact 7** will ensure that every state s supports the *de re* form of (2). (**Facts 6** and **7** are both proved in the appendix.) From a technical point of view, the crucial difference between names and descriptions is that, for most names n, $\mathcal{I}(n)(v)$ will be a constant function, whereas for most descriptions d, $\mathcal{I}(d)(v)$ will be a non-constant function (where v is any world). Thus, names will tend to behave rigidly with respect to metaphysical modals, while descriptions will tend to behave non-rigidly with respect to metaphysical modals.

4 Alternatives

4.1 Static two-dimensionalism?

The introduction of two-dimensionalism into this arena raises a further question: could a *static* version of two-dimensionalism solve our problems? Do we even need dynamic semantics if we already have two-dimensionalism?[14]

We can approach this question by noting that any theory of these matters will need to explain how it can be that definite descriptions are scopeless with respect to epistemic modals, but not scopeless with respect to metapysical modals. That is, they will need some way to explain why the *de re* form of (3) is inconsistent, while the *de re* form of (11) is assertable:

(3) The inventor of bitcoin might not be the inventor of bitcoin.
De re form: $(\lambda x.\Diamond_e x \neq b)(b)$

(11) The inventor of bitcoin might not have been the inventor of bitcoin.
De re form: $(\lambda x.\Diamond_m x \neq b)(b)$

In approaching this issue, we assume that the two-dimensionalist associates with the description 'the inventor of bitcoin' the following two-dimensional intension:

> $\mathcal{I}(b)$ is a function from pairs of worlds (v, v') to the inventor of bitcoin in v'.

The intension of this definite description needs to be sensitive to the second world of one of its arguments in order for the theory to predict that the description does not behave rigidly under metaphysical modals. We will assume that this holds generally for ordinary definite descriptions.

Consider the *de re* form of (3), $(\lambda x.\Diamond_e x \neq b)(b)$. Suppose the static two-dimensionalist is trying to evaluate this sentence for truth at a point of evaluation. The first step is presumably this:

$$[\![(\lambda x.\Diamond_e x \neq b)(b)]\!]^{c,v,v',g} = 1 \text{ iff}$$
$$[\![\Diamond_e x \neq b]\!]^{c,v,v',g'} = 1$$

[14] Thanks to an anonymous referee for raising this question.

where g' is an x-variant of g.

But what is $g'(x)$? In particular, what do variables range over? There are at least two options: (i) variables range over individuals, in which case $g'(x)$ is an individual, or (ii) variables range over individual concepts, in which case $g'(x)$ is an individual concept. The first option turns out to correspond to the approach discussed in §1, the approach that gave rise to the puzzle in the first place. The second option turns out to correspond to the individual concepts theory of §2, and thus fails for the same reason.

Consider the first option, according to which variables range over individuals. In that case $g'(x)$ is presumably $\mathcal{I}(b)(v, v')$, i.e. the inventor of bitcoin in world v'. But then this sentence is predicted to be true just in case:

> there is a world $v'' \in \mathcal{B}_c$ such that the inventor of bitcoin in world v' is distinct from the inventor of bitcoin in v''.

But assuming that 'the inventor of bitcoin' is non-rigid over $\mathcal{B}_c$—assuming we don't know who the inventor of bitcoin is—there will be such a world, as we discussed in §1. Thus, this approach will falsely predict that (3) is true in our context.

What about the second option which says that $g'(x)$ is an individual concept? If we take this option, we need to answer another question: which individual concept does $g'(x)$ denote? Two-dimensionalists typically distinguish between two individual concepts associated with a singular term, its *primary intension* and its *secondary intension* relative to a world. But for an ordinary definition description like the 'the inventor of bitcoin,' these simply coincide:

> The *primary intension* of b is a function that maps a world v' to $\mathcal{I}(b)(v', v')$, i.e. to the inventor of bitcoin at v'.
>
> The *secondary intension* of b at world v is a function that maps a world v' to $\mathcal{I}(b)(v, v')$, i.e. to the inventor of bitcoin at v' (here v is any world).

So let us suppose that $g'(x)$ is this individual concept, which we earlier called i_b.

Like the individual concepts approach in §2, this approach correctly predicts that (3) is inconsistent (false at every point of evaluation). The problem with this approach is what it predicts about the *de re* reading of (11), which should come out true. For on this approach, that formula presumably has the following truth-conditions at an arbitrary point of evaluation:

> $[\![(\lambda x.\Diamond_m x \neq b)(b)]\!]^{c,v,v',g} = 1$ iff
> $[\![\Diamond x \neq b]\!]^{c,v,v',g'} = 1$, where g' is just like g with the possible exception that $g'(x) = i_b$.

But then this is equivalent to the following:

> there is a world v'' such that $[\![x \neq b]\!]^{c,v,v'',g'} = 1$ iff
> there is a world v'' such that $[\![x]\!]^{c,v,v'',g'} \neq [\![b]\!]^{c,v,v'',g'}$ iff
> there is a world v'' such that $g'(x)(v'') \neq \mathcal{I}(b)(v, v'')$

But since $g'(x) = i_b$, $g'(x)(v'')$ is the inventor of bitcoin at v''. But $\mathcal{I}(b)(v, v'')$ is also the inventor of bitcoin at v''. Since there is no world v'' such that the inventor of bitcoin at v'' is distinct from the inventor of bitcoin at v'', there is no world v'' such that $g'(x)(v'') \neq \mathcal{I}(b)(v, v'')$, which means that the *de re* form of (11) is false at this point of evaluation. And since this point of evaluation was arbitrary, it follows that the *de re* form of (11) is inconsistent, which is not what we want.

Thus, neither of these versions of static two-dimensionalism replicate the results we achieved above with dynamic semantics.

4.2 Conclusion

Advocates of static two-dimensionalism might point out that there a third option that I have not considered: variables might be taken to range not over individuals or individual concepts, but over *two-dimensional intensions*, functions from pairs of worlds to individuals. I have not shown that such a view cannot be made to work, and I agree that it is an option worth exploring. But I leave such exploration as a task for future inquiry, since I am not aware of any extant theories of this kind, and adequately assessing this idea would require us to examine the consequences of such a move for other parts of the semantics (in particular, for the treatment of quantifiers). My own development of dynamic semantics here has built on an extant theory whose properties are relatively well-understood.

Furthermore, I should emphasize that the ambition of this essay was not to show that the puzzles I've been discussing *demand* a dynamic treatment. Rather, I hope to have established the more modest conclusion that a dynamic treatment of these problems *can* be given. This remains true even if a novel version of static two-dimensionalism is able to handle these data.

It's also worth pointing that static treatments other than a two-dimensional one might also be given. For example, *counterpart theory* might contain the resources for an alternative static treatment of these matters.[15] Answering the question of whether these topics are best treated within a dynamic system or within a static system would require us to examine these various possibilities. Since I can't hope to undertake that task here, let me close by mentioning two further issues that might be taken to bear on this question.

First, we have focused on one class of *de re*–*de dicto* distinctions, namely those involving singular terms. But of course the interaction of quantifiers and modals also give rise to *de re*–*de dicto* ambiguities, and so any full theory of these matters will have to encompass a theory of quantified epistemic and metaphysical modality.[16] Second, although we have been discussing both epistemic and metaphysical modals, we have not discussed their interaction in any detail.

[15] For relevant counterpart-theoretic approaches, see Ninan (2018) and Rabern (2018). For another static approach that may be relevant, see Mandelkern (2017, 2019).

[16] Quantified epistemic modality is discussed in Groenendijk et al. 1996, Aloni (2001) Beaver 2001, Yalcin 2015, Klinedinst and Rothschild 2016, Mandelkern 2017, 2019, Moss (2018), Ninan (2018), and Rabern (2018).

What happens when a metaphysical modal occurs within the scope of an epistemic modal, or vice-versa? This question may also bear on the choice between static and dynamic semantics.

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Appendix

One-dimensional dynamic semantics

Assume a language $\mathcal{L}$ of quantified multi-modal logic in which $=, \lambda, \neg, \wedge, \exists, \Diamond_e, \Diamond_m$ are the primitive logical symbols. The other logical symbols are defined in the usual way, e.g. $\forall x\phi$ is $\neg\exists x\neg\phi$. The language also contains individual constants, variables, and n-ary predicates. (What I called 'singular terms' in the text, will be represented here by individual constants.) Variables and individual constants are *terms*, and we use t, t', t_1, t_2, etc. as meta-variables over terms. The sentences (or formulas) of the language are defined in the usual way. The formation clause for the abstraction operator λ is as follows:

> If ϕ is a formula, x a variable, and a an individual constant, then $(\lambda x.\phi)(a)$ is a formula.

Definition 1. A *one-dimensional model for* $\mathcal{L}$ is a 4-tuple $\mathcal{M} = \langle \mathcal{W}, \mathcal{R}, \mathcal{D}, \mathcal{I} \rangle$, consisting of a non-empty set $\mathcal{W}$ (of *worlds*), a binary equivalence relation $\mathcal{R}$ on $\mathcal{W}$, a non-empty set $\mathcal{D}$ (of *individuals*), and an interpretation function $\mathcal{I}$ which assigns: to each individual constant a a function $\mathcal{I}(a)$ from worlds to individuals; and to each n-ary predicate P a function $\mathcal{I}(P)$ from worlds to elements of $\mathcal{D}^n$.

The definitions that follow ought to all be relativized to a model, but I suppress such relativization throughout the remainder of the discussion.

Definition 2. A variable assignment is a function from variables to individuals in $\mathcal{D}$. Given a variable assignment g and an individual o in $\mathcal{D}$, $g[x/o]$ is the x-variant of g such that $g[x/o](x) = o$.

Definition 3. A *possibility* is a pair of a world and a variable assignment. A *state of information* is a (possibly empty) set of possibilities.

To state the recursive semantics, the following notation is helpful:

Definition 4. Given a possibility $i = \langle v, g \rangle$, we have:

$i(x) = g(x)$ for any variable x

$i(a) = v(a) = \mathcal{I}(a)(v)$ for any individual constant a

$i(P) = \mathcal{I}(P)(v)$ for any predicate P

$i[x/o] = \langle v, g[x/o] \rangle$ for any individual $o \in \mathcal{D}$

$s[x/o] = \{i[x/o] : i \in s\}$

$i[x/a] = \langle v, g[x/i(a)] \rangle$

$s[x/a] = \{i[x/a] : i \in s\}$

$\mathcal{R}(i) = \{\langle v', g' \rangle : g' = g \text{ and } v\mathcal{R}v'\}$

The semantics can then be stated as follows:

Definition 5. *The update of state s by a sentence ϕ, $s[\phi]$*, is defined as follows:

$s[P(t_1, ..., t_n)] = \{i \in s : \langle i(t_1), ..., i(t_n) \rangle \in i(P)\}$

$s[t_1 = t_2] = \{i \in s : i(t_1) = i(t_2)\}$

$s[\neg\phi] = s - s[\phi]$

$$s[\phi \wedge \psi] = s[\phi][\psi]$$

$$s[(\lambda x.\phi)(a)] = \{i \in s : i[x/a] \in s[x/a][\phi]\}$$

$$s[\exists x\phi] = \{i \in s : \text{there is an } o \text{ in } \mathcal{D} \text{ s.t. } i[x/o] \in s[x/o][\phi]\}$$

$$s[\Diamond_m \phi] = \{i \in s : \mathcal{R}(i)[\phi] \neq \emptyset\}$$

$$s[\Diamond_e \phi] = \begin{cases} s & \text{if } s[\phi] \neq \emptyset; \\ \emptyset & \text{otherwise.} \end{cases}$$

The proofs of **Facts 1** and **2** (§3.1) are left to the reader. Recall **Fact 3**:

Fact 3. *If ϕ does not contain $\Diamond_m$, then for any state s and any individual constant a:*

$$s[(\lambda x.\Diamond_e \phi)(a)] = s[\Diamond_e \phi(a/x)]$$

This claim follows from the following more general claim, the proof of which is sketched below:

Fact 8. *If ϕ does not contain $\Diamond_m$, then for any state s and any individual constant a:*

$$s[(\lambda x.\phi)(a)] = s[\phi(a/x)]^{17}$$

Proof. The proof is by induction on the complexity of formulas, where the *complexity* of a formula is the number of logical symbols it contains (excluding identity). Here we discuss the base case and the cases for negation and the epistemic modal. Note that these are the cases most relevant to the formulas which we have been discussing.

Base case. Let $P(t_1, ..., t_n)$ be an arbitrary atomic formula, and let s be any state. We want to show:

$$s[(\lambda x.P(t_1, ..., t_n))(a)] = s[P(t_1, ..., t_n)(a/x)]$$

Let's use the following notation: for any of the t_i ($1 \leq i \leq n$):

$$t_i(a/x) = \begin{cases} a & \text{if } t_i \text{ is the variable } x; \\ t_i & \text{otherwise.} \end{cases}$$

So $P(t_1, ..., t_n)(a/x)$ is $P(t_1(a/x), ..., t_n(a/x))$. So we need to show:

$$s[(\lambda x.P(t_1, ..., t_n))(a)] = s[P(t_1(a/x), ..., t_n(a/x))]$$

Let i be any possibility. From the clauses for the abstraction operator and atomic formulas we have:

[17] As noted earlier, $\phi(a/x)$ is the result of replacing all free occurrences of x with a. The free occurrences of x in an atomic formula ϕ are all occurrences of x in ϕ. The free occurrences of x in a complex formula ϕ are those of its principal subformulas with the following exception: if ϕ is of the form $\exists x\psi$ or $(\lambda x.\psi)(a)$, then there are no free occurrences of x in ϕ.

$i \in s[(\lambda x.P(t_1, ..., t_n)(a)]$ iff
$i[x/a] \in s[x/a][P(t_1, ..., t_n)]$ iff
$\langle i[x/a](t_1), ..., i[x/a](t_n)\rangle \in i[x/a](P)$ iff
$\langle i[x/a](t_1), ..., i[x/a](t_n)\rangle \in i(P)$

And from the clause for atomic formulas, we have:

$i \in s[P(t_1(a/x), ..., t_n(a/x))]$ iff
$\langle i(t_1(a/x)), ..., i(t_n(a/x))\rangle \in i(P)$

So it will suffice to show that, for each of the t_i, $i[x/a](t_i) = i(t_i(a/x))$. So let t be any of the t_i. There are two cases: either t is x or t is not x.

First suppose that t is x. In that case $i[x/a](t) = i[x/a](x) = i(a)$. And since t is x, $t(a/x)$ is a. So $i(t(a/x)) = i(a)$. It follows that $i[x/a](t) = i(t(a/x))$.

Now suppose that t is not x. In that case, $i[x/a](t) = i(t)$. And since t is not x, $t(a/x)$ is t. So $i(t(a/x)) = i(t)$. So it again follows that $i[x/a](t) = i(t(a/x))$.

The case of formulas of the form $t_1 = t_2$ is simply the case where P is $=$, and $t_1, ..., t_n$ is t_1, t_2.

Induction step. Let n be a number greater than 0. Our induction hypothesis is that the result holds for every formula with complexity less than n. We want to show that the result holds for every formula ϕ with complexity n.

Negation. Suppose ϕ is $\neg\psi$ for some formula ψ that does not contain $\Diamond_m$. Let s be any state. We want to show:

$$s[(\lambda x.\neg\psi)(a)] = s[(\neg\psi)(a/x)].$$

Since the free variable occurrences in $\neg\psi$ are just those that occur in ψ, $(\neg\psi)(a/x)$ is $\neg\psi(a/x)$. So we want to show:

$$s[(\lambda x.\neg\psi)(a)] = s[\neg\psi(a/x)].$$

By the clause for negation, the induction hypothesis (IH), and the clause for the abstraction operator, we have:

(12) $s[\neg\psi(a/x)]$
$= s - s[\psi(a/x)]$
$= s - s[(\lambda x.\psi)(a)]$ (by IH)
$= s - \{i \in s : i[x/a] \in s[x/a][\psi]\}$
$= \{i \in s : i[x/a] \notin s[x/a][\psi]\}$.

Now, by the clauses for the abstraction operator and negation and (12), we have

$s[(\lambda x.\neg\psi)(a)]$
$= \{i \in s : i[x/a] \in s[x/a][\neg\psi]\}$
$= \{i \in s : i[x/a] \in (s[x/a] - s[x/a][\psi])\}$
$= \{i \in s : i[x/a] \in s[x/a]$ and $i[x/a] \notin s[x/a][\psi]\}$
$= \{i \in s : i[x/a] \notin s[x/a][\psi]\}$[18]
$= s[\neg\psi(a/x)]$ (by (12))

which is what we needed to show.

Epistemic possibility modal. Suppose now that ϕ is $\Diamond_e\psi$, for some formula ψ that does not contain $\Diamond_m$. Let s be any state. We want to show:

$$s[(\lambda x.\Diamond_e\psi)(a)] = s[(\Diamond_e\psi)(a/x)].$$

Note that $(\Diamond_e\psi)(a/x)$ is $\Diamond_e\psi(a/x)$. So we want to show:

$$s[(\lambda x.\Diamond_e\psi)(a)] = s[\Diamond_e\psi(a/x)].$$

Given the clause for the abstraction operator, it will suffice to show that, for any $i \in s$:

$i[x/a] \in s[x/a][\Diamond_e\psi]$ iff $i \in s[\Diamond_e\psi(a/x)]$[19]

Give the clause for the epistemic modal, this will hold just in case:

$s[x/a][\psi] \neq \emptyset$ iff $s[\psi(a/x)] \neq \emptyset$.

To establish this, suppose first that $s[x/a][\psi] \neq \emptyset$. In that case, there is an $i \in s$ such that $i[x/a] \in s[x/a][\psi]$. So by the clause for the abstraction operator, $i \in s[(\lambda x.\psi(a)]$. So by the induction hypothesis, $i \in s[\psi(a/x)]$, which means that $s[\psi(a/x)] \neq \emptyset$.

Now suppose $s[\psi(a/x)] \neq \emptyset$. So there is an $i \in s[\psi(a/x)]$. So by the induction hypothesis, $i \in s[(\lambda x.\psi(a)]$. By the clause for the abstraction operator, this implies that $i[x/a] \in s[x/a][\psi]$, which means that $s[x/a][\psi] \neq \emptyset$.

□

The following lemma is useful for establishing **Facts 4** and **5**:

Lemma 1. *Let $i = \langle v, g\rangle$ be any possibility in any state s. Then:*

$$i \in s[(\lambda x.\Diamond_m)(x \neq a)(a)]$$

just in case:

there is a v' such that $v\mathcal{R}v'$ and $v(a) \neq v'(a)$.

[18]Note that $i \in s$ iff $i[x/a] \in s[x/a]$.

[19]Here and elsewhere, we rely on the fact that the semantics possesses *the update property*, that is, that $s[\chi] \subseteq \chi$, for every state s and formula χ.

Proof. Let $i = \langle v, g\rangle$ be a possibility in any state s. By the clauses for the abstraction operator, negation, and identity, we have:

(13) $i \in s[(\lambda x.\Diamond_m x \neq a)(a)]$ iff
$i[x/a] \in s[x/a][\Diamond_m x \neq a]$ iff
$\mathcal{R}(i[x/a])[x \neq a] \neq \emptyset$ iff
there is an $i' \in \mathcal{R}(i[x/a])$ such that $i'(x) \neq i'(a)$ iff
there is a $\langle v', g'\rangle \in \mathcal{R}(i[x/a])$ such that $g'(x) \neq v'(a)$.

Let h be the x-variant of g such that $h(x) = v(a)$. So since $i = \langle v, g\rangle$, $i[x/a] = \langle v, h\rangle$. Given this, we have:

$$\mathcal{R}(i[x/a]) = \{\langle v', g'\rangle : g' = h \text{ and } v\mathcal{R}v'\}.$$

Since $h(x) = v(a)$, it follows that:

(14) there is a $\langle v', g'\rangle \in \mathcal{R}(i[x/a])$ such that $g'(x) \neq v'(a)$ iff
there is a $\langle v', g'\rangle$ such that $g' = h$, $v\mathcal{R}v'$, and $g'(x) \neq v'(a)$ iff
there is a v' such that $v\mathcal{R}v'$ and $h(x) \neq v'(a)$ iff
there is a v' such that $v\mathcal{R}v'$ and $v(a) \neq v'(a)$.

The result follows from (13) and (14). □

Fact 4. *Let s be a state such that, for each possibility i in s, a is non-rigid over $\mathcal{R}(i)$. Then:*

$$s[(\lambda x.\Diamond_m x \neq a)(a)] = s.$$

Proof. Let s be a state such that, for each i in s, a is non-rigid over $\mathcal{R}(i)$. Let i be an element of s. So if $i = \langle v, g\rangle$, there is a v' such that $v\mathcal{R}v'$ and $v(a) \neq v'(a)$. From **Lemma 1**, it follows that:

$$i \in s[(\lambda x.\Diamond_m x \neq a)(a)].$$

Since this holds for an arbitrary i in s, it holds for them all, from which our result follows. □

Fact 5. *Let s be a state such that, for each possibility i in s, a is rigid over $\mathcal{R}(i)$. Then:*

$$s[(\lambda x.\Diamond_m x \neq a)(a)] = \emptyset.$$

Proof. Let s be a state such that, for each i in s, a is rigid over $\mathcal{R}(i)$. Suppose, for *reductio*, that there is a possibility i such that:

$$i \in s[(\lambda x.\Diamond_m x \neq a)(a)].$$

Then if $i = \langle v, g \rangle$, it follows from **Lemma 1** that there is a v' such that $v\mathcal{R}v'$ and $v(a) \neq v'(a)$. But since $i \in s$, a is rigid over $\mathcal{R}(i)$, which means that, for all w, w' such that $v\mathcal{R}w$ and $v\mathcal{R}w'$, $w(a) = w'(a)$. Since $\mathcal{R}$ is an equivalence relation, $v\mathcal{R}v$. Since $v\mathcal{R}v'$, it follows that $v(a) = v'(a)$. Contradiction.

□

Two-dimensional dynamic semantics

The move to two-dimensional dynamic semantics requires us to slightly alter the definition of a model. A *two-dimensional model* for $\mathcal{L}$ is just like a one-dimensional model except that the interpretation function $\mathcal{I}$ assigns to each individual constant a a *two-dimensional intension* $\mathcal{I}(a)$, i.e. a function from pairs of worlds to individuals.

And as we noted in the body of the essay, a *possibility* is now a triple of a world, a world, and a variable assignment.

These changes necessitate some further changes to our notation:

Definition 6. Given a possibility $i = \langle v, v', g \rangle$, we have:

$$i(a) = \mathcal{I}(a)(v, v') \quad \text{for any individual constant } a$$

$$\mathcal{R}(i) = \{\langle w, w', g' \rangle : w = v, v'\mathcal{R}w', \text{ and } g' = g\}$$

The *statement* of the recursive semantics remains unchanged, though the clauses have a slightly different meaning owing to these changes in the underlying notation.

I said in the text that our solution to the initial two puzzles is preserved in the move from one-dimensional to two-dimensional dynamic semantics (§3.4). Consider the puzzle about definite descriptions, which requires us to reconcile the inconsistency of (3) and (7) with the assertability of (6).

Regarding the inconsistency of (3) and (7): For this, it suffices to note that **Fact 8** still holds in two-dimensional dynamic semantics. But we can also argue for the point directly, by showing the following:

Fact 9. *For any state s on any two-dimensional model $\mathcal{M}$:*

$$s[(\lambda x.\Diamond_e x \neq a)(a)] = \emptyset.$$

Proof. Suppose, for *reductio*, that i is in s and:

$$i[x/a] \in s[x/a][\Diamond_e x \neq a] \ .$$

Then, by the clauses for the epistemic modal, negation, and identity, there is an $j[x/a] \in s[x/a]$ such that $j[x/a](x) \neq j[x/a](a)$. But $j[x/a](x) = j(a)$, and $j[x/a](a) = j(a)$. So $j[x/a](x) = j[x/a](a)$. Contradiction.

□

Regarding the assertability of (6) in our context: Let σ be the set of worlds compatible with what we know. Then since we don't know whether not the CEO of Tesla is the inventor of bitcoin, there are worlds v in σ at which the

CEO of Tesla is the inventor of bitcoin, and worlds v' in σ at which the CEO of Tesla is not the inventor of bitcoin. So $\mathcal{I}(o)(v,v) = \mathcal{I}(b)(v,v)$, and $\mathcal{I}(o)(v',v') \neq \mathcal{I}(b)(v',v')$. Let s be a state the represents σ; so for some variable assignment g, $s = \{\langle w,w,g\rangle : w \in \sigma\}$. We argue that s supports (6). Given the recursive semantics, we have:

> $s[(6)] = s$ if there is an $i \in s$ such that $i(o) = i(b)$, and there is an $i \in s$ such that $i(o) \neq i(b)$

(Compare **Fact 2**.) This condition is met by our state s which represents σ: take i be to $\langle v,v,g\rangle$, take i' to be $\langle v',v',g\rangle$.

To establish **Facts 6** and **7**, the following lemma is useful:

Lemma 2. *For any possibility $i = \langle v,v',g\rangle$ in any state s:*

$$i \in s[(\lambda x.\Diamond_m)(x \neq a)(a)]$$

just in case:

> *there is a w' such that $v'\mathcal{R}w'$ and $\mathcal{I}(a)(v,v') \neq \mathcal{I}(a)(v,w')$.*

Proof. Let $i = \langle v,v',g\rangle$ be any possibility in any state s. By the clauses for the abstraction operator, negation, and identity, we have:

(15) $i \in s[(\lambda x.\Diamond_m x \neq a)(a)]$ iff
$i[x/a] \in s[x/a][\Diamond_m x \neq a]$ iff
$\mathcal{R}(i[x/a])[x \neq a] \neq \emptyset$ iff
there is an $i' \in \mathcal{R}(i[x/a]$ such that $i'(x) \neq i'(a)$ iff
there is a $\langle w,w',g'\rangle \in \mathcal{R}(i[x/a])$ such that $g'(x) \neq \mathcal{I}(a)(w,w')$.

Let h be the x-variant of g such that $h(x) = \mathcal{I}(v,v')(a)$. So $i[x/a] = \langle v,v',h\rangle$. Given this, we have:

$$\mathcal{R}(i[x/a]) = \{\langle w,w',g'\rangle : w = v, g' = h, \text{ and } v'\mathcal{R}w'\}$$

Since $h(x) = \mathcal{I}(a)(v,v')$, we have:

(16) there is a $\langle w,w',g'\rangle \in \mathcal{R}(i[x/a])$ such that $g'(x) \neq \mathcal{I}(a)(w,w')$ iff
there is a $\langle w,w',g'\rangle$ such that $w = v, g' = h, v'\mathcal{R}w'$, and $g'(x) \neq \mathcal{I}(a)(w,v'')$ iff
there is a w' such that $v'\mathcal{R}w'$ and $h(x) \neq \mathcal{I}(a)(v,w')$ iff
there is a w' such that $v'\mathcal{R}w'$ and $\mathcal{I}(a)(v,v') \neq \mathcal{I}(a)(v,w')$.

The result follows from (15) and (16). $\square$

Fact 6. *Let a be an individual constant such that, for any world v, $\mathcal{I}(a)(v)$ is a constant function from worlds to individuals. Then for any state s:*

$$s[(\lambda x.\Diamond_m x \neq a)(a)] = \emptyset$$

Proof. Let a be an individual constant such that, for any world v, $\mathcal{I}(a)(v)$ is a constant function from worlds to individuals. Suppose, for *reductio*, that $i \in s[(\lambda x.\Diamond_m x \neq a)(a)]$. Let $i = \langle v, v', g\rangle$. By **Lemma 2**, there is a w' such that $v'\mathcal{R}w'$ and $\mathcal{I}(a)(v, v') \neq \mathcal{I}(a)(v, w')$. But then $\mathcal{I}(a)(v)$ is not a constant function. Contradiction. □

Fact 7. *Let a be an individual constant such that, for any world v, $\mathcal{I}(a)(v)$ is not a constant function. Then for any state s:*

$$s[(\lambda x.\Diamond_m x \neq a)(a)] = s$$

Proof. We stated **Fact 7** assuming that $\mathcal{R}$ was universal, an assumption we retain in this proof. Assume that, for any world v, $\mathcal{I}(a)(v)$ is not a constant function. Let $i = \langle v, v', g\rangle$ be any possibility in any state s. We want to show:

$$i \in s[(\lambda x.\Diamond_m)(x \neq a)(a)]$$

Given **Lemma 2**, this will hold just in case:

there is a w' such that $v\mathcal{R}w'$ and $\mathcal{I}(a)(v, v') \neq \mathcal{I}(a)(v, w')$.

Since $\mathcal{R}$ is universal, this will hold just in case:

there is a w' such that $\mathcal{I}(a)(v, v') \neq \mathcal{I}(a)(v, w')$.

This condition must hold because $\mathcal{I}(a)(v)$ is not a constant function. Thus, $i \in s[(\lambda x.\Diamond_m)(x \neq a)(a)]$, which is what we needed to show. □